Math 114, Prep 11.10 (Taylor Series)

Name: ______________________________ **Section:** ________

Names of collaborators: ______________________________

Main Points:

1. Taylor series and Taylor polynomials
2. using Taylor series

Recall that we can consider a power series as polynomial with an infinite number of terms. For example,

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$$

We can also consider this as representing a *family of polynomial approximations*:

$$\begin{aligned}
\frac{1}{1-x} &\approx s_0(x) = 1 \\
\frac{1}{1-x} &\approx s_1(x) = 1 + x \\
\frac{1}{1-x} &\approx s_2(x) = 1 + x + x^2 \\
\frac{1}{1-x} &\approx s_3(x) = 1 + x + x^2 + x^3 \\
&\vdots \\
\frac{1}{1-x} &\approx s_n(x) = 1 + x + x^2 + x^3 + \ldots + x^n \\
&\vdots
\end{aligned}$$

Recall (from Calc I), that we have another way of obtaining polynomial approximations: via derivatives. If a function $f(x)$ is continuous at $x = a$, we can approximate $f(x)$ by a constant function $f(x) \approx f(a)$, at least near $x = a$. If $f(x)$ is differentiable, we can approximate it by a linear function (the tangent line).

$$\begin{aligned}
f(x) &\approx f(a) \\
f(x) &\approx f(a) + f'(a)(x-a)
\end{aligned}$$

Extending this idea leads to the notion of Taylor polynomials and Taylor series. (See Example 1.)

Exercises.

1. Read pages 753-755 of the section. State Theorem 5 (page 754).

2. Read Examples 4 and 5.

   (a) What is the Taylor series for $\sin(x)$ centered at $x = 0$? (A Taylor series centered at $x = 0$ is also called a Maclaurin series.) What is the Taylor series of $\cos(x)$ centered at $x = 0$?

   (b) Write out the first three Taylor polynomials of $\sin x$ at $x = 0$. Do the same for $\cos x$.

3. Find the first four Taylor polynomials of $f(x) = \sin(2x)$ centered at $x = 0$.

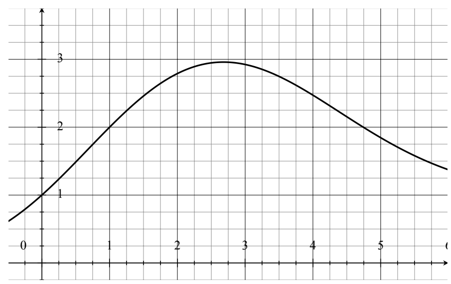
4. (a) Find the first four Taylor polynomials of $g(x) = \frac{1}{x}$ centered at $x = -3$.

   (b) Find the Taylor series of $f(x)$ centered at $x = -3$. What is the radius of convergence?

5. We can use known Taylor series to find Taylor series for related functions. (See Example 6.) Use the Taylor series for e^x to find a Taylor series for $e^{-x/2}$.

6. We can also use known Taylor series to find the sums of certain series. (See Example 10.) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n\, n!}$.

7. The graph of a function $f(x)$ is shown below.



(a) Explain why the series

$$2 - 0.8(x-1) + 0.4(x-1)^2 - 0.1(x-1)^3 + \dots$$

is *not* the Taylor series of f centered at $x = 1$.

(b) Explain why the series

$$2.8 + 0.5(x-2) + 1.5(x-2)^2 + 0.1(x-2)^3 + \dots$$

is *not* the Taylor series of f centered at $x = 2$.