Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Basic use of IBP
2. Two tricks

1. Basic use of IBP

Integration by parts is a way to use the "reverse product rule" to exchange a hard integral for an easier one. Here is an example:

What is $\frac{d}{dx}\,(x\,\sin x)$? (Use the product rule.)

Given your answer above, what is $\int(\sin x + x\cos x)\,dx$?

On the other hand, notice that we can split the integral above into two integrals:

$$\int(\sin x + x\cos x)\,dx \;=\; \int \sin x\,dx \;+\; \int x\cos x\,dx \qquad (*)$$

The first of these two integrals is easy:

$\int \sin x\,dx \;=$

Since we know two out of three integrals in the equation $(*)$, we can determine the third integral simply by subtracting.

$\int x\cos x\,dx \;=$

The integration by parts rule is a generalization of what we have just done. Recall that the product rule can be written as:

$$\frac{d}{dx}\,u(x)\,v(x) \;=\; u'(x)\,v(x) \;+\; u(x)\,v'(x)$$

Restating in terms of integrals and rearranging gives:

$$\int u(x)\,v'(x)\,dx \;=\; u(x)\,v(x) \;-\; \int u'(x)v(x)\,dx$$

Using the shorthand $du = u'(x)\,dx$ and $dv = v'(x)\,dx$, we can rewrite this as:

$$\boxed{\int u\,dv \;=\; uv \;-\; \int v\,du}$$

See Example 1, page 465, "Solution using Formula 2," for a solution of the example using this formula.

Tip: IBP is a good strategy to try when the integrand is a product of two functions. In order for IBP to work, you need to be able to differentiate one of the functions and anti-differentiate the other. Choose u to be the function you want to differentiate and v' to be the function you want to anti-differentiate.

Exercises

1. Evaluate the integral using integration by parts with the indicated choices of u and dv. Make sure to state explicitly what v and du are. (See Example 1, page 465, "Solution using Formula 2.")

   (a) $\int x^2 \ln(x)\, dx; \quad u = \ln x, \quad dv = x^2\, dx$

   (b) $\int \theta \cos(2\pi\theta)\, d\theta; \quad u = \theta, \quad dv = \cos(2\pi\theta)\, d\theta$

2. Evaluate the integrals.

   (a) $\int y\, e^{2y}\, dy$

(b) $\int t^2 \sin t \, dt$ (Hint: Use IBP twice.)

2. Two tricks

Trick #1 Sometimes IBP can be used even when the integrand does not look like a product of two functions. In particular, if we know the derivative of the integrand, we can let the whole integrand be u and we can let $v' = 1$. See Example 2, page 465.

Exercise

3. Evaluate the integral: $\int \arctan x \, dx$.

Trick #2 Sometimes IBP can be used even when neither part of the integrand becomes simpler when differentiated, if we can notice a pattern of repeating derivatives. See Example 4, page 466.

4. Evaluate the integral: $\int e^{\theta} \cos\theta \, d\theta$

   Hint: Use IBP twice. You will get: $\int e^{\theta} \cos\theta \, d\theta = e^{\theta}\sin\theta + e^{\theta}\cos\theta - \int e^{\theta}\cos\theta d\theta$. Notice that the original integral shows up on the right side of the equation! Call the original integral $\mathcal{I}$. Then we have an equation: $\mathcal{I} = e^{\theta}\sin\theta + e^{\theta}\cos\theta - \mathcal{I}$. Solve this equation algebraically to find $\mathcal{I}$.