Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Trig substitution: for some integrands with $x^2 + a^2$, $\sqrt{a^2 - x^2}$, or $\sqrt{x^2 - a^2}$
2. Transform to trig. integral and use the methods of 7.2

When the integrand looks similar to the derivative of an inverse trig function it is sometimes possible to transform the integral to a trig. integral. Then we can use the methods of 7.2 to evaluate the integral.

For these problems, it is important to remember the Pythagorean theorem as well as right triangle trigonometry (SOH CAH TOA).

Exercises

1. Consider the integral: $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$.

   First we construct an appropriate triangle. Let $\text{HYP} = 2$ and $\text{OPP} = x$. Then $\text{ADJ} = \sqrt{4-x^2}$.(Good!) Draw and label the triangle below.

   Using the triangle: $\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{x}{2}$, so

   $x =$

   $dx =$

   In addition, from the triangle, we can see $\frac{\sqrt{4-x^2}}{2} = \frac{\text{ADJ}}{\text{HYP}} = \cos\theta$, so

   $\sqrt{4-x^2} =$

Now rewrite the integral using trig functions of θ and $d\theta$. Simplify as much as possible; then integrate.

Finally, use your triangle to rewrite your answer in terms of x.

2. Consider the integral: $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-9}}$.

First we construct an appropriate triangle. Let HYP $= x$ and ADJ $= 3$. Then OPP $= \sqrt{x^2-9}$.(Good!) Draw and label the triangle below.

Using the triangle: $\sec\theta = \frac{\text{HYP}}{\text{ADJ}} = \frac{x}{3}$, so

$x =$

$dx =$

In addition, from the triangle, we can see $\frac{\sqrt{x^2-9}}{3} = \frac{\text{OPP}}{\text{ADJ}} = \tan\theta$, so

$\sqrt{x^2-9} =$

Now rewrite the integral using trig functions of θ and $d\theta$. Simplify as much as possible; then integrate.

Finally, use your triangle to rewrite your answer in terms of x.

3. Use an appropriate trig substitution to evaluate the integral. Draw and label an appropriate triangle.

$$\int \sqrt{1-4x^2}\, dx$$

Hint: You may want to use the identity $\sin(2\theta) = 2\sin\theta\,\cos\theta$.

4. Use an appropriate trig substitution to evaluate the integral. Draw and label an appropriate trangle.

$$\int_0^1 \frac{dx}{(x^2+1)^2}$$

Hint: Here you want $\textsc{opp} = x$ and $\textsc{adj} = 1$ so that $(\textsc{hyp})^2 = x^2 + 1$.

Note: Be careful with the limits of integration! Just as with simple substitution, you have two choices: (1) transform the limits of integration when you transform the integral or (2) break the problem into two steps, finding the indefinite integral first.

5. Which substitution should you try in each case?

When you see $\sqrt{a^2 - x^2}$, try ...

When you see $\sqrt{x^2 - a^2}$, try ...

When you see $x^2 + a^2$, try ...