Math 114, Prep 6.1 (Areas Between Curves)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Using vertical distance between curves
2. Using horizontal distance between curves

1. Using vertical distance between curves

Recall that we use a definite integral to find the (signed) area between a curve and the x-axis. If $f(x) \geq 0$ on an interval $[a, b]$, then the definite integral gives a literal area:

$$\text{(area between } f(x) \text{ and } x\text{-axis from } x = a \text{ to } x = b) \quad = \quad \int_a^b f(x)\, dx$$

Similarly, if a function $f(x) \geq g(x)$ on an interval $[a, b]$, the area between the two curves from $x = a$ to $x = b$ is obtained by subtracting the smaller area from the greater area:

$$\text{(area between } f(x) \text{ and } g(x) \text{ from } x = a \text{ to } x = b) \quad = \quad \int_a^b f(x)\, dx \; - \int_a^b g(x)\, dx \quad = \quad \int_a^b \big(f(x) - g(x)\big)\, dx$$

Exercises.

1. Find the area bounded by the curves: $y = \sin x$, $y = e^x$, $x = 0$, $x = \pi/2$. (Hint: You need to figure out which curve lies above the other. Sketching the graphs will help. See Example 1, p 423.)

2. Find the area bounded by the curves $y = x^2 - 4x$ and $y = 2x - x^2$. (Hint: You need to find the points of intersection of the two curves, as in Example 2, p 423.)

3. Find the area bounded by the curves $y = \sin(\pi x/2)$ and $y = x$. (Hint: These curves cross each other three times, and so it is necessary to break up the area into two areas, as in Example 5, p 425. The two paragraphs before Example 5 are helpful to read.)

2. Using horizontal distances between curves

Not all curves can be expressed in the form $y = f(x)$. For example, a right-opening parabola has equation $x = y^2$. The area between two curves like this can sometimes be expressed using an integral in y instead of an integral in x. In particular, if the curve $x = R(y)$ is to the right of a curve $x = L(y)$ for y between the values of $y = c$ and $y = d$,

$$(\text{area between } R(y) \text{ and } L(y) \text{ from } y = c \text{ to } y = d) \quad = \quad \int_c^d R(y) - L(y)\, dx$$

See the discussion and example (Example 6) on page 426.

Exercises

4. Find the area bounded by the curves $x = y^2 - 4y$ and $x = 2y - y^2$ from $y = 1$ to $y = 2$. (Sketching the two curves will help.)

5. Find the area bounded by the curves $4x + y^2 = 12$ and $x = y$. (Hint: rewrite the first equation to get x on a side by itself.)