**Name:** ______________________ **Section:** ________

**Names of collaborators:** ______________________

**Bring your laptops to class! We will be using *Mathematica*!**

## Main Points:

1. areas inside/outside of curves, between curves, etc.
2. Use *Mathematica* to plot

## 1. Area inside a polar curve

When using polar coordinates, it is important to notice that the "boxes" in "polar graph paper" are not actually rectangular, and that they get larger and larger the farther away from the origin they are.

In Cartesian coordinates, our infinitessimal area is $dA = y\,dx$ (for a vertical rectangle) or $dA = x\,dy$ for a horizontal rectangle. We might expect our polar infinitessimal area to be $dA \overset{??}{=} r\,d\theta$. However, this is *not* the case, because in polar coordinates, our $dA$ is not the area of a rectangle at all, it is the area of a sector!

If a sector has radius $r$ and angle $d\theta$, its area is a fraction of a circle, and the fraction is determined by the ratio of $d\theta$ to $2\pi$, the "angle" of a whole circle:

$$dA \;=\; (\text{area of circle}) \times \frac{d\theta}{2\pi} \;=\; (\pi r^2) \times \frac{d\theta}{2\pi} \;=\; \tfrac{1}{2}\, r^2 d\theta$$

Thus the area between a polar curve $r = r(\theta)$ and the origin, bounded by the rays $\theta = \theta_1$ and $\theta = \theta_2$ is:

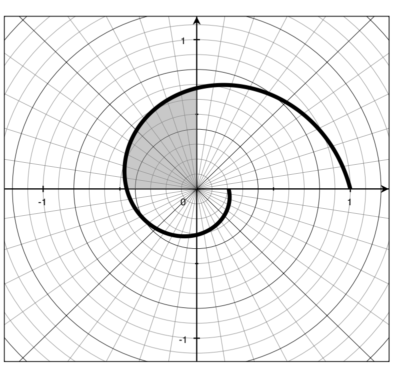
$$A \;=\; \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 \, d\theta$$

To find the area between two polar curves, find the area inside the outer curve and the area inside the inner curve separately, and then subtract.

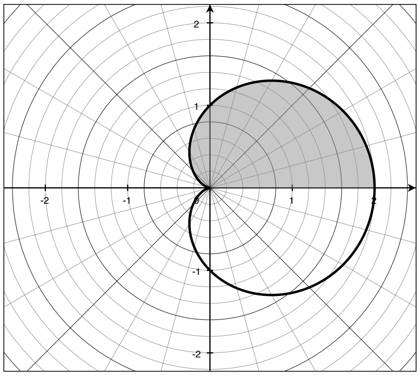
In general, it can be difficult to set up the integral for the area enclosed by polar curves or between polar curves, because polar curves can wrap around the origin more than once, and because there are a number of subtleties involved in finding the intersection points of polar curves, due to the fact that there is more than one way to write the same point in polar coordinates.

**Exercises.**

1. Find the area of the region bounded by $r = e^{-\theta/4}$ in the sector $\pi/2 \le \theta \le \pi$.



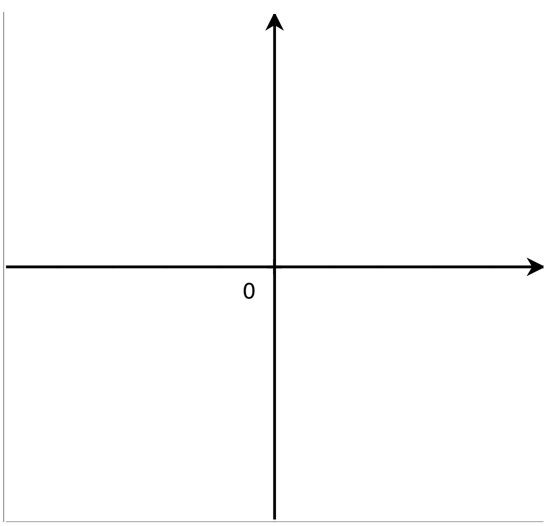
2. Find the area of the region bounded by the curve $r = 1 + \cos\theta$ and the $x$-axis.



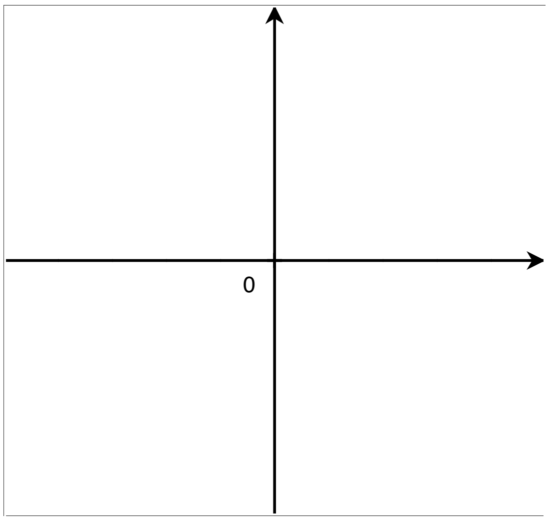

## 2. Using *Mathematica* to help set up the integrals

Recall that to plot a curve in polar coordinates using *Mathematica*, we use the `PolarPlot` command. For example to plot $r = 2\cos\theta$ from $\theta = 0$ to $8\pi$, use: `PolarPlot[2*Cos[theta], {theta, 0, 8*Pi}]`

3. Use *Mathematica* to plot the curve $r^2 = \sin(2\theta)$, sketch the curve below, and find the area it encloses. See Example 1. (Hint: Use symmetry!)



4. Find the area of the region inside $r = 1 - \sin\theta$ but outside $r = 1$. See Example 2. You may use *Mathematica* to help you sketch the curves.



5. Find the area inside both $r = \sin 2\theta$ and $r = \sin\theta$. You may use *Mathematica* to help you sketch the curves. (Hint: First divide the region into two symmetric regions $\mathcal{R}_1$ and $\mathcal{R}_2$; you will find the area of $\mathcal{R}_2$ and then multiply by two. To find the area of $\mathcal{R}_1$, you will need to divide *it* into two regions as well ...)