Name: ______________________ **Section:** __________

Names of collaborators: ______________________

Please prepare Part 1 for class. We will complete Part 2 together in class.

Main Points:

1. sequences: various notations
2. convergence/divergence of sequences

1. Sequences

A sequence is a list of numbers in a definite order. There are several ways of denoting a sequence, but the simplest involves curly braces and elipses. Some examples of sequences denoted in this way are:

$$\{1, 2, 3, 4, 5, 6, \dots\} \qquad \{1, -1, 1, -1, 1, -1, \dots\} \qquad \{1,\ 1/2,\ 1/4,\ 1/8, \dots\}$$

Another way of denoting a sequence is by giving a formula for the n^{th} term of the sequence. For example the three sequences above could be represented with the following three formulas:

$$a_n = n\ , \ \ n \geq 1 \qquad b_n = (-1)^n\ , \ \ n \geq 0 \qquad c_n = 1/2^n\ , \ \ n \geq 0$$

See Examples 1 and 2.

Exercises.

1. The formula for the n^{th} term of a sequence is given. Use the formula to find the first five terms of the sequence, and write the sequence in the notation with curly braces and elipses.

   (a) $a_n = \dfrac{n}{n+1}\ , \ \ n \geq 2$

   (b) $b_n = \dfrac{2n}{n^2+1}\ , \ \ n \geq 0$

   (c) $a_n = \dfrac{(-1)^n\, n}{n!+1}\ , \ \ n \geq 1$ (Hint: remember that $n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$.)

2. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

   (a) $\{\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \dots\}$

   (b) $\{4, -7, 10, -13, 16, \dots\}$

   (c) $\{6, 9/4,\ 4/3,\ 15/16, 18/25, 21/36 \dots\}$

Some sequences are more easily described **recursively**. See Example 3(c) and the beginning of Example 14.

Exercises

3. List the first five terms of the following recursively defined sequences.

   (a) $a_1 \;=\; 6, \quad a_{n+1} \;=\; a_n/n$

   (b) $b_0 \;=\; 1, \quad b_1 \;=\; 2, \quad b_{n+1} \;=\; 2b_{n-1} + b_n$

2. The Limit of a Sequence

Informally, if the numbers a_n approach a specific, finite number L as $n \to \infty$, then the sequence is said to **converge**, and L is called the **limit** of the sequence. If a sequence does not have a limit, it is said to **diverge**. Read pages 692-693 for a more careful discussion of the limit of a sequence.

Exercises.

4. There are several useful theorems we can use for finding limits of sequences.

   (a) Copy down Theorem 3 (page 693) and explain how it is used in Example 6 (page 694).

   (b) Copy down Theorem 6 (page 694) and explain how it is used in Example 8.

   (c) Copy down the Squeeze Theorem (box at the top of page 694) and explain how it is used in Example 10.

   (d) Read Example 11, and summarize the result (stated in the box at the end of the example.)

5. Determine whether the following sequences converge or diverge. If convergent, find the limit.

   (a) $a_n = \dfrac{3+5n^2}{n+n^2}$ (Hint: divide top and bottom by the highest power of n in the denominator.)

   (b) $a_n = e^{2n/(n+2)}$ (Hint: what happens to the "inside function"?)

   (c) $a_n = \dfrac{3^{n+1}}{5^n}$ (Hint: find the common factor.)

   (d) $a_n = 2^{-n}\cos(n\pi)$ (Hint: Take the absolute value.)

   (e) $a_n = n\sin(1/n)$ (Hint: Find a matching function $f(x)$ and use L'Hospital's rule.)