Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. basic definitions: series, partial sums, sum of series, convergence/divergence
2. geometric series, harmonic series
3. test for divergence

1. Series

Consider a sequence $\{a_1, a_2, a_3, \dots\}$. A **series** is what we use to try to determine the accumulation of these values:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

Of course, there is no guarantee that this actually represents a finite number. To make this more precise we need to look at a new sequence, the sequence of partial sums.

Read pages 703-705, up to and including the definition in the red box.

Exercises.

1. Consider the series

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$$

(a) Calculate the first several partial sums:

$$s_1 = \frac{1}{3} =$$

$$s_2 = \frac{1}{3} + \frac{1}{9} =$$

$$s_3 = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} =$$

$$s_4 = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} =$$

(b) Find a formula for the n^{th} term of the series.

$$a_n =$$

(c) Write the series in sigma notation.

2. Geometric Series and Harmonic Series

Two very important examples are geometric series and harmonic series.

Exercises.

2. Read Example 2.

   (a) Summarize the results of Example 2 below. (See the box in the middle of page 706.)

   (b) The series in Exercise 1, above, is a geometric series. Is it convergent? If so, what is its sum?

3. Consider the following geometric series:

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \dots$$

   (a) Find the common factor (denoted r in Example 2.)

   (b) Write the series in sigma notation.

   (c) Does the series converge? If so, what is its sum?

4. Consider the following geometric series:

$$10 - 2 + 0.4 - 0.08 + \dots$$

   (a) Find the common factor.

   (b) Write the series in sigma notation.

   (c) Does the series converge? If so, what is its sum?

5. Read Example 8.

   (a) Write the harmonic series below.

   (b) Use *Mathematica* to make the plots in Figure 3 yourself.

```
aPoints = Table[{n, 1/n}, {n, 1, 100}];
sPoints = Table[{n, Sum[1/k, {k, 1, n}]}, {n, 1, 100}];
ListPlot[{aPoints, sPoints}, PlotStyle -> {Red, Blue}]
```

   (c) The red dots represent the sequence of partial sums of the harmonic series. Does it look like the series converges or diverges?

   (d) According to Example 8, does the harmonic series converge or diverge?

3. Test for Divergence

A sequence whose terms do not approach zero has no chance of converging. This is formalized in Theorem 6 and the Test for Divergence.

Exercises

6. Find the Test for Divergence on page 709 and write it below.

7. Consider the series

$$2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \dots$$

   (a) Write a formula for the n^{th} term, a_n, of this series.

   $a_n \quad =$

   (b) Write the series in sigma notation.

   (c) Find the limit of the terms of the series:

   $\lim_{n\to\infty} a_n \quad =$

   (d) Apply the Test for Divergence to show that the series diverges.