Math 114, Prep 11.2B (Finding the Sum of a Series)

Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. evaluating the limit of partial sums
2. combinations of convergent series

1. Evaluating the Limit of Partial Sums

Recall that the sum of a series is the limit of partial sums (if the limit exists), i.e.

$$S = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{n=1}^{N} a_n$$

Usually, it is very difficult to find a formula for the N^{th} partial sum of a series, but in a few cases it can be done: geometric series (as in Example 2) and telescoping series (as in Example 7).

Exercises.

1. Consider the series

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$$

(a) Find a formula for the n^{th} term of the series. Write it in the form $a_n = a \cdot r^{n-1}$.

$a =$ $r =$

$a_n =$

(b) Reread Example 2. Use Equation 3 (near the top of page 706) to write a formula for the N^{th} partial sum.

$S_N =$

(c) Evaluate the limit of partial sums, if the limit exists:

$\lim_{N\to\infty} S_N =$

(d) What is the sum of the series?

2. Consider the series $1 - 1 + 1 - 1 + \ldots$.

   (a) Find a formula for the n^{th} term a_n of the series.

   $$a_n \quad =$$

   (b) Write the series in sigma notation.

   (c) Find the first four partial sums of the series.

   (d) Find a formula for the N^{th} partial sum of the series:

   $$S_N \quad = \quad \begin{cases} & \text{if } N \text{ is} \\ & \text{if } N \text{ is} \end{cases}$$

   (e) Evaluate the limit of partial sums, if the limit exists:

   $$\lim_{N\to\infty} S_N \quad =$$

   (f) What is the sum of the series?

3. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$. (See Example 7.)

(a) Use a partial fraction decomposition to rewrite the terms of the series in the form:

$$\frac{2}{n^2+4n+3} = \frac{A}{n+1} + \frac{B}{n+3}$$

i.e. find suitable A and B.

(b) Write out, but do not calculate $S_1, \ldots S_4$. (See Example 7.)

$S_1 =$

$S_2 =$

$S_3 =$

$S_4 =$

(c) Now cancel terms to "simplify" but not calculate $S_1, \ldots, S_4$.

$S_1 =$

$S_2 =$

$S_3 =$

$S_4 =$

(d) Find a formula for the N^{th} partial sum of the series:

$S_N =$

(e) Evaluate the limit of partial sums, if the limit exists:

$$\lim_{N\to\infty} S_N \quad =$$

(f) What is the sum of the series?

2. Combinations of Convergent Series

Theorem 8 describes legitimate manipulations of convergent series. This can be useful for finding the sum of a series that can be rewritten as a sum, difference, or constant multiple of a known convergent series.

Exercises.

4. State Theorem 8. (Make sure to include the words, not just the formulas!)

5. Explain Note 4 (bottom of page 710) in your own words.

6. Determine whether the series is convergent or divergent. If convergent, find its sum.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3^{n-1}} + \frac{3}{2^{n-1}}\right)$

(b) $\displaystyle\sum_{n=3}^{\infty}\left(\frac{2}{3^{n-1}} + \frac{3}{2^{n-1}}\right)$

(c) $\dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{9} + \dots$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{3+(-1)^n}{2(3^n)}$