Math 114, Prep 11.4 (Comparison Test)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. comparing with familiar series, whose convergence/divergence is known
2. making a careful argument and invoking the Comparison Test

When our intuition tells us that a certain series ought to converge (or diverge) because it is similar to a familiar series, one that we *know* converges (resp. diverges), we can sometimes use the Comparison Test to give a careful argument in support of our intuition.

For example, we may *suspect* that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$$

converges, since it is similar to a p-series with $p = 2$. We will use the Comparison Test to justify this.

Note: Recall that we can use the Integral Test to determine whether or not a series converges; it is not necessary to have an intuition ahead of time about whether or not the series converges. In contrast, to use the Comparison Test, you must first have a hunch about whether the series converges or diverges. The Comparison Test is then used to give a careful argument (hopefully) proving your hunch to be correct.

Exercises.

1. Read pages 722-723.

   (a) State the Comparison Test (bottom of page 722.)

   (b) To be able to use the Comparison Test, it is helpful to have some familiar series to compare with. Give two examples of simple families of series whose convergence/divergence is known to us. (See the paragraph after the proof of the Comparison Test.)

2. In this exercise, we will use the Comparison Test to show that the series $\sum_{i=1}^{\infty} \frac{1}{n^2+n+1}$ converges, by comparison with the p-series, with $p = 2$.

   (a) (**Set-up**) Define the series $\sum a_n$ to be the given series and $\sum b_n$ to be the other (more familiar) series, whose convergence/divergence is known.

   (b) (**Check Hypotheses**) Check to make sure the series $\sum a_n$ and $\sum b_n$ satisfy the hypothesis in the Comparison Test, namely that they are series with positive terms.

   (c) (**Compare Terms**) Prove an inequality for the terms of the two series. (For convergence, show $a_n \leq b_n$ for all n, or, for divergence, show $b_n \leq a_n$ for all n).

   (d) (**Discuss known series**) Explain how you know that the more familiar series $\sum b_n$ converges or diverges.

   (e) (**Apply Test and Draw Conclusion**) Use the Comparison Test to conclude that the series $\sum a_n$ converges (or diverges), and state your conclusion in a sentence. Make sure to include the phrase "by the Comparison Test" somewhere in your sentence.

3. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^3+4}$.

   (a) Do you think this series converges or diverges? (It converges.) What known (convergent) series can you compare it to? Can you prove an inequality like $a_n \le b_n$ for the terms (a_n) of this series and the terms (b_n) of the familiar (convergent) series?

   (b) Give a careful argument, using the Comparison Test, to prove that the series converges. (Your argument should follow the outline given in the previous problem.)

4. Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{1+\sin(n)}{10^n}$.

   (a) Do you think this series converges or diverges? What known series can you compare it to? Can you prove an inequality of terms?

(b) Give a careful argument, using the Comparison Test, to prove that the series converges (or diverges.)

5. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n - 1/2}$.

(a) Do you think this series converges or diverges? What known series can you compare it to? Can you prove an inequality of terms?

(b) Give a careful argument, using the Comparison Test, to prove that the series converges (or diverges.)