Math 114, Practice for Exam 2

Name: ____

Instructions: The exam will have eight problems. Make sure that your reasoning and your final answers are clear. Include labels and units when appropriate. No notes, books, or calculators are permitted during the exam. The following formulas will be provided.

$$\begin{aligned} \sin^2 x &= \tfrac{1}{2}(1-\cos 2x) \\ \cos^2 x &= \tfrac{1}{2}(1+\cos 2x) \\ \sin x\, \cos x &= \tfrac{1}{2}\sin 2x \end{aligned}$$

$$\begin{aligned} \int \sec x\,dx &= \ln|\sec x + \tan x| & \int \tan x\,dx &= \ln|\sec x| \\ \int \csc x\,dx &= \ln|\csc x - \cot x| & \int \cot x\,dx &= \ln|\sin x| \end{aligned}$$

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1. **Basic Facts and Concepts**

   Fill in the blanks question.

   (a) Average value of a function, its graphical interpretation as the height of a certain rectangle, the MVT for integrals.

   (b) Formulas for volume (dV) of an infinitessimal disc and washer.

   (c) Interpretation of integral as accumulation, in concrete situations.

   (d) Parametric curves: how to eliminate a parameter.

   (e) How to convert from polar coordinates to Cartesian coordinates and vice versa.

   (f) Formulas for parametric and polar calculus (finding first and second derivatives with respect to x, finding areas, etc.)

   (*See also the Chapter Review Concept Checks: CR6 #2, 6, CR10 #2, 4, 5ab, CR11 2*)

2. **Average value of a function and the MVT for Integrals.**
   Consider the function $f(x) = 2x - x^2$ on the interval $[0, 2]$.
   (a) Find the average value of f on the given interval.
   (b) Find c such that $f_{\text{ave}} = f(c)$.
   (c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

   (*See also 6.5 #1-15, 19, 21.*)

3. **Areas and Volumes (Cartesian Coordinates)**

   (a) Sketch the region bounded by the given curves, and find the area.

   $$y = x^2, \quad y = 4x - x^2$$

   (b) Find the volumes of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the following lines: (i) the x-axis, (ii) the y-axis, (iii) $y = 2$.

   (*See also Ch 6 Review #2-6, 12-14.*)

4. **Sample Application of the Integral: Consumer Surplus**

The demand function $p(x)$ is the price that a company has to charge in order to sell x units of a commodity. If X is the amount of the commodity that is currently available (the sales level), then $P = p(X)$ is the current selling price. The consumer surplus $\int_0^X \big(p(x) - P\big)\,dx$ is the amount of money saved by consumers in purchasing the commodity at price P corresponding to an amount demanded of X.

The demand function for can openers, in dollars, is given by $p = 450/(x+8)$.

(a) Find the consumer surplus when the selling price is \$10.

(b) What does the area under the curve $y = p(x)$ from $x = 0$ to $x = 37$ represent? Use a complete sentence.

(c) What does the area under the curve $y = 10$ from $x = 0$ to $x = 37$ represent?

(d) What does the area between the curves $y = p(x)$ and $y = 10$ from $x = 0$ to $x = 37$ represent?

(*See also 8.4 #4-10, CR 8 #17.*)

5. **Parametric Calculus**

   (a) Consider the parametric curve:

   $$x(t) \;=\; 1 + \ln(t) \qquad y(t) \;=\; t^2 + 2$$

      i. Find the tangent line to the curve at the point $(1, 3)$, without eliminating the parameter.
      ii. Now eliminate the parameter, and verify that your answer to (a) is correct by finding the slope of the tangent line again.

   (b) Consider the parametric curve

   $$x(t) \;=\; t^2 + 1 \qquad y(t) \;=\; t^2 + t$$

   For which values of t is the curve concave up?

   (*See also 10.2 #1-20*)

6. **Polar Calculus**

(a) Find the slope of the tangent line to the curve $r = 4\sin\theta$ when $\theta = \pi/6$.

(b) Sketch the curve $r = 4\sin\theta$ and find the area it encloses.

(c) Find the area of the region that lies inside $r = 4\sin\theta$ and outside $r = 2$.

(*See also 10.3 #15-20, 55-64, 10.4 # 9-12, 23-25, 29-31*)