Math 114, Practice for Exam 3

Name: ______________________________

Instructions: The exam will have five problems. Make sure that your reasoning and your final answers are clear. Include labels and units when appropriate. No notes, books, or calculators are permitted during the exam. The following results will be provided.

Remainder Estimate for Integral Test: $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_{n}^{\infty} f(x)dx$

Alternating Series Estimation Theorem: $|R_n| = |s - s_n| \leq |a_{n+1}|$

---

1. **Basic Facts and Concepts**

   Fill in the blanks question.

   (a) Geometric sequences: $a_n = ar^n$ (which of these converge; when convergent, what is the limit)
   (b) Geometric series: $\sum ar^n$ (which of these converge; when convergent, what is the limit)
   (c) Harmonic series and p-series: $\sum \frac{1}{n^p}$ (which of these converge)
   (d) Limit of terms of series vs. limit of partial sums of series
   (e) Statement of tests for convergence and divergence (know the hypotheses!)
   (f) Absolute and conditional convergence

   (*See also the CR Concept Check: 4 and True/False Quiz: 1, 7, 8, 9, 11, 12, 17, 18, 21*)

2. **Sequences vs. Series**

   For each sequence below, write out the first five terms. Then find the limit of the sequence, if it exists. Does the associated series converge or diverge? Justify your answer with an appropriate convergence test.

   (a) $a_n = \dfrac{2n-1}{3n+1}$, $n \geq 1$
   (b) $b_n = n^2 e^{-n}$, $n \geq 0$

   (*See also 11.1 # 3-8, CR #1-4, 6.*)

3. **Finding the Sum of a Series.**

   Find the sum of the series, if it exists.

   (a) $\displaystyle\sum_{n=2}^{\infty} 16 \left(\frac{-3}{4}\right)^n$
   (b) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$
   (c) A series $\displaystyle\sum_{n=1}^{\infty} a_n$ for which $s_n = \displaystyle\sum_{k=1}^{n} a_k = \frac{7+3n}{n}$.

   (*See also CR #27.*)

4. **Convergence Tests**

Determine whether the following series converge or diverge. In the case of convergence, state whether the convergence is conditional or absolute. Make sure that all of your conclusions are well-supported with careful arguments.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{9n^3+n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,2^n}{n!}$

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k}{1+2^k}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(*See also CR #11, 13-16, 20-21, 23-26.*)

5. **Estimating the Sum of a Series**

(a) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$. The first ten partial sums (rounded to two decimal places) are given in the table below.

n	1	2	3	4	5	6	7	8	9	10
S_n	1.00	1.25	1.36	1.42	1.46	1.49	1.51	1.53	1.54	1.55

One of your classmates conjectures that the sum of the series (rounded to two decimal places) is 1.66. Use the Remainder Estimate for the Integral Test to explain why this is impossible.

(b) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$. The first nine partial sums (rounded to three decimal places) are given in the table below.

n	1	2	3	4	5	6	7	8	9
S_n	1.000	0.750	0.861	0.799	0.839	0.811	0.831	0.816	0.828

One of your classmates conjectures that the sum of the series (rounded to three decimal places) is 0.817. Use the Alternating Series Estimation Theorem to explain why this is impossible.

(*See also 11.3 #36(a)(d), 37(a)(d), 11.5 #23-26.*)

6. **Power Series**

For each power series, write out the first three partial sum functions, and find the interval of convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{n-1}}{5^{n-1}\,n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{4^n\,n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$

(*See also 11.8 #3-12.*)