Name: ______________________________ **Section:** ________

Names of collaborators: ______________________________

Main Points:

1. Use *Mathematica* to graph partial sum functions.
2. Notice the significance of radius of convergence.

Exercises.

1. Recall that $\sum_{n=1}^{\infty} x^n$ is a function with domain $(-1, 1)$. Using the formula for the sum of a convergent geometric series,

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \qquad (\text{ for } -1 < x < 1)$$

(a) Write out the first five partial sum functions. (The first three are done for you.)

$s_0(x) = 1$

$s_1(x) = 1 + x$

$s_2(x) = 1 + x + x^2$

$s_3(x) =$

$s_4(x) =$

What is the degree 10 partial sum function?

$s_{10}(x) =$

(b) Use *Mathematica* to plot $f(x)$ and each of the partial sum functions in (a) on a common set of axes. Start by plotting $f(x)$ and $s_0(x)$ together:

```
Plot[{1/(1-x), 1}, {x, -2, 2}, PlotRange -> {-6, 6}]
```

Note that this command sets the x-limits of the viewing rectangle to be -2 and 2, and it sets the y-limits of the viewing rectangle to be -6 and 6. You can change the viewing rectangle by changing the x or y-limits in the `Plot` command.

Then plot $g(x)$ and $s_1(x)$ together:

```
Plot[{1/(1-x), 1+x}, {x, -2, 2}, PlotRange->{-6, 6}]
```

Do the same for each of the partial sum functions $s_2(x), \ldots, s_4(x)$, as well as $s_{10}(x)$.

(c) For what x-values does $s_0(x)$ give a good approximation of $f(x)$? What about $s_1(x)$? $s_2(x)$? etc. Fill in the table to show the range of x-values for which $s_n(x)$ approximates $f(x)$ well. Do your answers make sense, given the domain of the power series?

n	x-range for s_n
0	
1	
2	
3	
4	
10	

2. Consider the function given by the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)(x-1)^n}{2^{n+1}}$.

(a) What is the center of this power series? a = __________

(b) Use the Ratio Test to find the radius of convergence of the series.

(c) Write out the first five partial sum functions. (The first three are done for you.)

$$s_0(x) = \tfrac{1}{2}$$

$$s_1(x) = \tfrac{1}{2} + \tfrac{1}{2}(x-1)$$

$$s_2(x) = \tfrac{1}{2} + \tfrac{1}{2}(x-1) + \tfrac{3}{8}(x-1)^2$$

$$s_3(x) =$$

$$s_4(x) =$$

What is the degree 10 partial sum function? Use the `Sum` command in *Mathematica* to help you.

```
Sum[((n+1)(x-1)^n)/(2^(n+1)), {n, 0, 10}]
```

$$s_{10}(x) =$$

(d) In Section 11.9, we will learn how to show that this power series agrees with the function $g(x) = 2/(3-x)^2$ on its interval of convergence. Use *Mathematica* to plot $g(x)$ and each of the partial sum functions in (a) on a common set of axes. Start by plotting $g(x)$ and $s_0(x)$ together:

```
Plot[{2/(3-x)^2, 1/2}, {x, -2, 4}, PlotRange -> {-2, 5}]
```

Then plot $g(x)$ and $s_1(x)$ together:

```
Plot[{2/(3-x)^2, 1/2+(1/2)(x-1)}, {x, -2, 4}, PlotRange -> {-2, 5}]
```

Do the same for each of the partial sum functions $s_2(x), \ldots, s_4(x)$, as well as $s_{10}(x)$.

(e) For what x-values does $s_0(x)$ give a good approximation of $g(x)$? What about $s_1(x)$? $s_2(x)$? etc. Fill in the table to show the range of x-values for which $s_n(x)$ approximates $g(x)$ well. What do you notice? Do your observations make sense with your answers for (a) and (b)?

n	x-range for s_n
0	
1	
2	
3	
4	
10	

3. Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.

   (a) Write out the first four partial sum functions. (The first two are done for you.) Use the `Sum` command in *Mathematica* to help you. For a number N, to find $s_N(x)$,

   `Sum[(-1)^n x^(2n+1)/(2n+1)!, {n, 0, N}]` (*Fill in a number for* `N`.)

   $s_0(x) \;=\; x$

   $s_1(x) \;=\; x - \frac{1}{6}x^3$

   $s_2(x) \;=$

   $s_3(x) \;=$

   (b) Plot $s_{10}(x)$ on the interval $[-6, 6]$. Instead of writing out $s_{10}(x)$, use the sum command:

   `Plot[Sum[(-1)^n x^(2n+1)/(2n+1)!, {n, 0, 10}], {x, -6, 6}]`

   The graph should look like the graph of a familiar function. Which one?

4. (**Bonus** +3) The Bessel function of order one is $\displaystyle J_1(x) \;=\; \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{n!\,(n+1)!\,2^{2n+1}}$.

   (a) Show that its domain is $(-\infty, \infty)$, using the Ratio Test.

   (b) The function $J_1(x)$ is `BesselJ[1,x]` in *Mathematica*. Plot partial sums along with $J_1(x)$ in *Mathematica* to determine the smallest degree N such that $s_N(x)$ gives a good approximation for $J_1(x)$ on $[-6, 6]$.

   ```
   Plot[{BesselJ[1,x], Sum[(-1)^n x^(2n+1)/(n!(n+1)!2^(2n+1)), {n,0,N}]}, {x,-6,6}]
   ```

   $N \;=$

5. Print off your *Mathematica* work, and staple it to this packet. Make sure that all of the graphs you were asked to produce are shown in your print-off. (You should have thirteen or fourteen graphs: six for #1, six for #2, one for #3, and, optionally, one for #4.)