Math 114, Sample Quiz 1

Name: Amy Section: ______

You have 20 minutes to complete the following problems, without using your notes, book, or calculator.

Part 1: Trigonometry

1. Fill in the following table, using the five standard angles in the first quadrant.

Angle, θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
deg	rad			
0	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	undef.

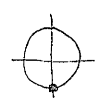
2. State the three Pythagorean Trig Identities:

(a) $\sin^2\theta + \cos^2\theta = 1$

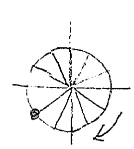
(b) $\tan^2\theta + 1 = \sec^2\theta$

(c) $1 + \cot^2\theta = \csc^2\theta$

3. $\cos(270^\circ) =$ 0



4. $\cot\left(-\frac{5\pi}{6}\right) =$ $\sqrt{3}$



$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{-\sqrt{3}/2}{-1/2}$

Part 2: Derivatives

5. Differentiate.

   (a) $\frac{d}{dx}\ 3x^7 - x^5 + x^{-1} \ = \ 21x^6 - 5x^4 - x^{-2}$

   (b) $\frac{d}{dx}\ 2\sec x \ = \ 2\tan x \sec x$

   (c) $\frac{d}{dx}\ \arcsin(x) \ = \ \frac{1}{\sqrt{1-x^2}}$

   (d) $\frac{d}{dx}\ 2^x - 2^\pi \ = \ \ln(2) \cdot 2^x$

6. Find the derivative, without simplifying afterward, and write your answer in the space provided.

   (a) $f(x) = x^8 e^{5x}$

   $f'(x) = (8x^7)(e^{5x}) + (x^8)(5e^{5x})$

   Product Rule

   (b) $g(t) = \dfrac{\ln t}{2t-1}$

   $g'(t) = \dfrac{(1/t)(2t-1) - (\ln t)(2)}{(2t-1)^2}$

   Quotient Rule

   (c) $y = \arctan(x^2)$

   $\dfrac{dy}{dx} = \left(\dfrac{1}{1+(x^2)^2}\right)(2x)$

   Chain Rule

Part 3: Antiderivatives

7. Find an antiderivative for the given function.

(a) $f(x) = \sec^2 x$

$$F(x) = \tan x + 72$$

(b) $J(s) = \dfrac{1}{s\sqrt{s^2-1}}$

$$\mathcal{L}(s) = \operatorname{arcsec}(s) - 21$$

(c) $Q(t) = \dfrac{t^2-t+1}{t^2} = 1 - \frac{1}{t} + \frac{1}{t^2} = 1 - \frac{1}{t} + t^{-2}$

$$P(t) = t - \ln|t| + \left(\tfrac{1}{-1}\right)t^{-1} + \tfrac{11}{4}$$

$$P(t) = t - \ln|t| - 1/t + 11/4$$

8. Evaluate the indefinite integrals:

(a) $\int x^3 - x^{-3}\,dx$

$$= \tfrac{1}{4}x^4 + \tfrac{1}{2}x^{-2} + C$$

(b) $\int \cos(w) + \cos(\pi)\,dw$

$$= \sin(w) + \cos(\pi)\cdot w + C = \sin(w) - w + C$$

(c) $\int \sqrt{t}(t-1)\,dt$

$$= \int \sqrt{t}\cdot t - \sqrt{t}\ dt = \int t^{3/2} - t^{1/2}\ dt$$

$$= \tfrac{2}{5}t^{5/2} - \tfrac{2}{3}t^{3/2} + C$$

$$= 2t^{3/2}\left(\tfrac{1}{5}t - \tfrac{1}{3}\right) + C$$

$$= 2t\sqrt{t}\left(\tfrac{1}{5}t - \tfrac{1}{3}\right) + C$$

Part 4: Definite integrals and substitution

9. Find the signed area between the curve $y = \sqrt{x}$ and the x-axis from $x = 0$ to $x = 4$.

$$A = \int_0^4 \sqrt{x}\, dx = \int_0^4 x^{1/2}\, dx = \frac{2}{3} x^{3/2} \Big|_0^4 =$$

$$= \frac{2}{3} x\sqrt{x} \Big|_0^4 = \frac{2}{3}(4 \cdot 2 - 0) = \frac{16}{3}$$

10. Evaluate the indefinite integrals:

(a) $\int 2x \sqrt[3]{x^2+1}\, dx$ $\qquad u = x^2 + 1 \qquad du = 2x\, dx$

$$= \int \sqrt[3]{u}\, du = \int u^{1/3}\, du = \frac{3}{4} u^{4/3} + C = \frac{3}{4} \sqrt[3]{(x^2+1)^4} + C$$

(b) $\int t^4 \sin(t^5)\, dt$ $\qquad u = t^5 \qquad du = 5t^4\, dx$

$$= \frac{1}{5} \int 5t^4 \sin(t^5)\, dt = \frac{1}{5} \int \sin u\, du = -\frac{1}{5} \cos u + C = -\frac{1}{5} \cos(t^5) + C$$

(c) $\int \cos^3\theta \sin\theta\, d\theta$ $\qquad u = \cos\theta \qquad du = -\sin\theta\, d\theta$

$$= -\int (\cos\theta)^3 (-\sin\theta\, d\theta) = -\int u^3\, du = -\frac{1}{4} u^4 + C = -\frac{1}{4} \cos^4\theta + C$$

(d) $\int \frac{w}{1-w}\, dw$ $\qquad u = 1 - w \qquad du = -dw$

$\qquad w = 1 - u$

$$= -\int \frac{1-u}{u}\, du = -\int \frac{1}{u} - 1\, du = -(\ln|u| - u) + C = -\ln|1-w| + (1-w) + C$$

$$= -w - \ln|1-w| + C$$

(new constant)