Logistical Information

- 1:30 pm - 3:30 pm Mon Dec 19, in our usual classroom
- Most problems will be similar to problems on homework and previous exams.
- There will be one fill-in-the-blank question with 30-40 blanks.
- No calculators, notes, books, cell phones permitted.
- Bring whatever you need to help yourself concentrate for 2 hrs: watch, water bottle, granola bar ...

The final exam is cumulative.

- Consult your review sheets for Exams 1, 2, and 3 for lists of basic facts and formulas to know, topics to know, and review problems for Units 1, 2, and 3.
- Also use the problems from Exams 1, 2, and 3 for practice.

Basic Facts and Formulas To Know from Unit 4:

- Sum of finite geometric series $a + ax + \cdots + ax^n$, sum of infinite geometric series $a + ax + ax^2 + \ldots$
- Harmonic series: what it is and that it diverges.
- Convergence/divergence of series of the form $\Sigma\, 1/n^p$ (p-series).

Topics from Unit 4: Series, Power Series, Applications of Series

- Geometric Series: partial sum formula, convergence/divergence, sum of convergent geometric series, applications involving geometric series (9.2)
- Series: difference between a series and a sequence, difference between the convergence of a series and the convergence of the sequence of its terms, the sum of the series as the limit of partial sums (9.3)
- Power Series: ratio test, radius of convergence (9.4, 9.5)
- Taylor Series: finding Taylor series using derivatives, finding Taylor series by substitution or multiplication, using Taylor series in applications (10.2, 10.3)

Additional Practice Problems for Unit 4:

Ch 9 Rev: 1, 2, 6, 7, 9, 12, 21, 22, 23, 37, 42, 43, 45, 49, 56, 58-61, 72, 73, 75, 80, 95, 97
Ch 10 Rev: 1-4, 6, 7, 11-18, 23, 24, 26, 30, 33, 36, 37, 44, 45, 47, 49

1. Fill in the blanks.

When $|x| < 1$, the geometric series $\displaystyle\sum_{n=0}^{\infty} ax^n$ converges to ______________ .

For p __________ , the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges.

Suppose that $\{S_n\}$ is the sequence of partial sums for the series $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\lim_{n\to\infty} S_n = 10$. Then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ ______________________________ . (*diverges* or *converges to* ...)

If $r = 1$, the sequence $\{r^n\}$ ________________________ (*diverges* or *converges to* ...), and

the series $\displaystyle\sum_{n=1}^{\infty} a\, r^{n-1}$ ________________________ (*diverges* or *converges to* ...).

The harmonic series, $\displaystyle\sum_{n=1}^{\infty}$ __________ , is an example of a ____________________ (*convergent* or *divergent*) series.

Suppose the sum of the series $\displaystyle\sum_{n=0}^{\infty} a_n$ is 3, and let S_n denote the n^{th} partial sum.

Then $\displaystyle\lim_{n\to\infty} a_n =$ __________ and $\displaystyle\lim_{n\to\infty} S_n =$ __________ .

Suppose $\displaystyle\sum_{n=0}^{\infty} a_n = 3$, and let S_n denote the n^{th} partial sum of the series. Then the limit of the terms of the series is $\displaystyle\lim_{n\to\infty} a_n =$ __________ and the limit of the partial sums of the series is $\displaystyle\lim_{n\to\infty} S_n =$ __________ .

Suppose the Taylor series for a function $g(x)$ centered at $x = 0$ is: $2 - x + 2x^2 - x^3 + 2x^4 - x^5 + \dots$

Then $g(0) =$ ______ , $g'(0) =$ ______ , and $g''(0) =$ ______ .

2. Let $a_n = \dfrac{1}{n} - \dfrac{1}{n+1}$.
   (a) Find the limit of the sequence $\{a_n\}$.
   (b) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$.
      i. List the first three partial sums s_1, s_2, s_3 of the series.
      ii. Find a closed formula for s_n.
      iii. Find the sum of the series.

3. Dr. LaValle drinks two cups of coffee per day, but she would like to gradually wean herself off of coffee, so she decides to reduce her coffee intake by 10% each day, starting tomorrow.
   (a) How much coffee does Dr. LaValle drink today? tomorrow? in n days?
   (b) How much coffee does Dr. LaValle drink, in total, over the first three days of her coffee intake reduction scheme?
   (c) How much coffee does Dr. LaValle drink, in total, over the first n days of her coffee intake reduction scheme?
   (d) What is the maximum amount of coffee that Dr. LaValle will drink over the course of the rest of her life, starting today?

4. Find the center and radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{5^n\, n^2}$.

5. Estimate $\int_0^1 \cos(x^2)\, dx$ using a Taylor polynomial with three nonzero terms.