Math 114, *Mathematica* Lab

Name: ———————————————————————————— **Section:** —————————

Names of collaborators: ————————————————————————————————

Due at the beginning of class Monday November 27.

This lab contains instructions for using *Mathematica* to answer questions about Cartesian curves, parametric curves, and sequences. The commands you need are introduced one by one. Most of the parts of the problems below simply tell you to enter a command in *Mathematica*. However, some parts of some problems also ask you to answer a question, so make sure to answer all the questions! When you have completed the lab, print off your *Mathematica* work, and staple it to this packet.

General *Mathematica* tips:

1. **Capital Letters** Every built-in function (like `Plot`, `Sqrt`, `Sin`) and every built-in constant (like `Pi`) begins with a capital letter.

2. **Parentheses** Use parentheses to specify the order of operations, as when using a graphing calculator.

3. **Square Brackets** Whenever passing an argument to a function (i.e. plugging something in to a function), you must use square brackets. For example, use `Sin[Pi/4]` to find $\sin(\pi/4)$.

4. **Curly Braces** Whenever creating lists or ranges, use curly braces. For example, when using the `Plot` command, use `{x, -10, 10}` to set the x-range to $[-10, 10]$.

5. **Shift + Enter** To execute a command, it is necessary to press `Shift + Enter`.

I. Cartesian Curves.

1. Tables and graphs for $f(x) = x^2$.

   (a) Use the following command to define the function $f(x)$ in *Mathematica*.

   > `f[x_] := x^2`

   (After you type the command in make sure to press `Shift + Enter`. You won't see anything happen because you are simply defining something, not asking for an output of any kind.)

   (b) Use the `Table` command to create a list of (x, y) pairs of the form $(x, f(x))$.

   > `Table[{x,f[x]}, {x, -4, 4}]`

   (You should get a list of (x, y) pairs as an output when you press `Shift + Enter`.)

   (c) Use the `TableForm` command to create a two-column input-output table for $f(x)$.

   > `TableForm[Table[{x,f[x]}, {x, -4, 4}]]`

   (d) Use the `Plot` command to obtain a graph of $f(x)$ on the x-range $[-4, 4]$ and y-range $[-5, 15]$.

   > `Plot[f[x], {x, -4, 4}, PlotRange -> {-5, 15}]`

2. A tangent line for $f(x) = x^2$.

    (a) Use the following command to find the derivative of $f(x)$:

        `f'[x]`

    (b) Use the following command to find the local linearization of $f(x)$ at $x = 2$.

        `f[2] + f'[2](x-2)`

    According to *Mathematica*, the local linearization is: _____

    (c) Imitate 1(a) to define a function $L(x)$ that is the local linearization of $f(x)$ at $x = 2$.

    (d) Use the `Plot` command to produce graphs of $f(x)$ and $L(x)$ on the same set of axes:

        `Plot[{f[x], L[x]}, {x, -4, 4}, PlotRange -> {-5, 15}]`

    Notice that, because we are plotting a pair of functions, the two functions are enclosed in curly braces (because we always use curly braces for lists.)

3. Area, arclength, and integration with $f(x) = x^2$.

    (a) Find an antiderivative for $f(x)$ using the `Integrate` command.

        `Integrate[f[x], x]`

    Note that you need to specify the variable of integration.

    (b) Use the `RegionPlot` command to produce a picture of the region under the curve $y = x^2$ and above the x-axis, between $x = -4$ and $x = 4$.

        `RegionPlot[y < f[x] && y >0 , {x, -4, 4}, {y, -5, 15}]`

    Estimate the area of the region using your picture and a simple hand calculation. (One reasonable thing to do would be to approximate the two regions with triangles.)

        $A \approx$ _____

    (c) Use the `Integrate` command to find the exact area under the curve:

        `Integrate[f[x], {x, -4, 4}]`

    According to *Mathematica*, the exact area under the curve is: A = _____

    (d) Using your graph of $f(x)$, estimate the arclength of the curve $y = f(x)$ from $x = -4$ to $x = 4$, using a simple hand calculation. (One reasonable thing to do would be to approximate with two straight lines.)

        $\ell \approx$ _____

(e) Use the `Integrate` command to find the exact arclength of the curve:

```
Integrate[Sqrt[1+(f'[x])^2], {x, -4, 4}]
```

According to *Mathematica*, the exact arclength is: ℓ = _____

(f) Use the `NIntegrate` command to find a numerical approximation for arclength of the curve. (The command is the same as the previous, except use the numerical integration command `NIntegrate` instead of the symbolic integration command `Integrate`.)

According to *Mathematica*, the approximate arclength is: $\ell \approx$ _____

II. Parametric Curves.

4. Tables and graphs for $x = 3\cos t$, $y = 2\sin t$.

   (a) Use the syntax described in 1(a) to define functions $x(t)$ and $y(t)$. Remember to capitalize the first letters for the `Sin` and `Cos` commands, and remember to press `Shift + Enter`.

   (b) Use `ParametricPlot` to graph the parametric curve:

   ```
   ParametricPlot[{x[t], y[t]}, {t, 0, 2*Pi}]
   ```

   What is the shape of the parametric curve? _____

   (c) (Challenge) A parametrization of the tangent line to the curve when $t = \pi/4$ is given by the linearizations of $x(t)$ and $y(t)$ at $t = \pi/4$. Graph the parametric curve and the tangent line on a single set of axes using `ParametricPlot`.

   (d) Use the `Integrate` command to find the area enclosed by the parametric curve. (Hint: Find the area enclosed by the curve in the first quadrant and multiply by four.) Remember that to find the area enclosed by a parametric curve from $x = a$ to $x = b$, you use an integral of the form

   $$\int_{t=\alpha}^{t=\beta} y(t)\, x'(t)\, dt \qquad \text{where } x(\alpha) = a \text{ and } x(\beta) = b$$

   According to *Mathematica*, the area enclosed by the curve is: A = _____

   (e) Use the `NIntegrate` command to find the arclength of the parametric curve. Remember that the integral for arclength along a parametric curve from $t = \alpha$ to $t = \beta$ is:

   $$\int_{\alpha}^{\beta} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

   According to *Mathematica*, the approximate arclength is: $\ell \approx$ _____

III. Sequences

5. Tables and graphs of the sequence $a_k = \ln(k)/k$.

   (a) Define the sequence $a_k = \ln(k)/k$ using the following command:

   ```
   a[k_] := Log[k]/k
   ```

   (b) To plot the sequence, we will first create a table of pairs (k, a_k) using the `Table` command:

   ```
   Table[{k, a[k]}, {k, 1, 20}]
   ```

   (c) Make an input-output table of these pairs using the `TableForm` command. Use the `N` command to get numerical approximations for a_k. (So, instead of `a[k]`, use `N[a[k]]`.)

   (d) Use `ListPlot` to plot the pairs (k, a_k).

   ```
   ListPlot[Table[{k, a[k]}, {k, 1, 20}], PlotRange -> {0, 0.4}]
   ```

   Can you tell, from your graph, what the limit of the sequence is (if it even exists?)   _____

   (e) Use the `Limit` command to find the limit of the sequence:

   ```
   Limit[a[k], k -> Infinity]
   ```

   According to *Mathematica* the limit of the sequence is $\lim\limits_{k \to \infty} \ln(k)/k$ =   _____

   (f) Use `ListPlot` to graph the pairs (k, a_k) with $1 \le k \le 100$.

   Does the limit found in (e) seem more plausible now?   _____

6. Summing the terms of the sequence $a_k = \ln(k)/k$.

   (a) Define a partial sum function $s_n = a_1 + \cdots + a_n = \sum\limits_{k=1}^{n} a_k$ with the command:

   ```
   s[n_] := Sum[a[k], {k, 1, n}]
   ```

   (b) Create a list of pairs (n, s_n) using the `Table` command for $1 \le n \le 20$.

   (c) Use the `ListPlot` command to plot the pairs (n, s_n).

   Does it look like the sequence s_n has a finite limit?   _____

7. Print off your *Mathematica* work, and staple it to this packet. Make sure that all of the commands you were asked to enter into *Mathematica* are shown in your print-off.