1. Determine whether the series converges or diverges. (To justify each of your conclusions, cite a general fact, and show how it applies to the specific example.) If the series converges, find the sum.

   (a) $\displaystyle\sum_{n=0}^{\infty} 5(-1/3)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} (-3)(1/2)^n$ (c) $\displaystyle\sum_{n=0}^{\infty} (-3/2)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

2. For each expression below, state whether it represents a sequence or a series. Then determine whether it converges or diverges. Justify your conclusions carefully.

   (a) $(-1)^n,\ n \geq 1$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n$ (c) $\displaystyle\frac{1}{\sqrt{n}},\ n \geq 1$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

   (e) $\displaystyle\frac{n^2-1}{2n^2},\ n \geq 1$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{2n^2}$ (g) $\displaystyle\frac{2n+1}{n^2+n},\ n \geq 1$ (h) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n}$

3. Find the center and radius of convergence of the power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n\, n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{4^n\, n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$

4. Find the fourth-degree Taylor polynomial centered at $x = a$ for the function $f(x)$.

   (a) $f(x) = \sin(x),\ a = \pi/2$ (b) $f(x) = \ln(x),\ a = 1$ (c) $\displaystyle f(x) = \frac{1}{2+x},\ a = 1$

5. In this problem, use the known Taylor series for e^x, $\sin x$, and $\cos x$, around $x = 0$.

   (a) Find the Taylor series for the following functions about $x = 0$, using known Taylor series:

   (i) $\displaystyle\frac{\sin x}{x}$ (ii) $\cos(x^2)$ (iii) $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ (iv) $\displaystyle\frac{e^x - 1}{x}$

   (b) Evaluate the following limits using the appropriate Taylor series, and check your work using L'Hospital's Rule.

   (i) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$ (ii) $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$

   (c) Find the Taylor series for the sine integral function, $\mathrm{Si}(t) = \int_0^t \frac{\sin x}{x}\, dx$, centered at $t = 0$.

   (d) Estimate the integral using the first three nonzero terms of the Taylor series: $\int_0^1 \cos(x^2)\, dx$.