Math 114, Taylor Polynomials (Section 10.1)

Name: ______________________________ **Section:** __________

Read Section 10.1, focusing on pages 538-539. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Taylor Polynomials of Degree 1: Linear Approximation.** Reread Example 1.
   (a) What is the Taylor polynomial of degree 1 (i.e. the linear approximation) for $g(x) = \cos x$, with x in radians, for x near zero?
   (b) Try this: What is the Taylor polynomial of degree 1 for $h(x) = e^x$ for x near zero?

2. **Taylor Polynomials of Degree 2: Quadratic Approximation.** Reread Example 2.
   (a) What is the quadratic approximation to $g(x) = \cos x$ for x near zero?
   (b) Try this: What is the quadratic approximation to $h(x) = e^x$ for x near zero?

3. **Higher Degree Taylor Polynomials.** Reread Examples 3 and 4 (bottom of page 541, top of page 542).
   (a) What is the Taylor polynomial of degree 8 for $g(x) = \cos x$ for x near zero?
   (b) Which Taylor polynomial, P_2 or P_8 is a better approximation for $g(x)$ near zero? Explain.

4. **Taylor Polynomials around $x = a$.** Reread Example 7.

   (a) What is the Taylor polynomial of degree 4 approximating $f(x) = \ln x$ for x near 1?

   (b) Look at the graph of $P_4(x)$ and the graph of $\ln(x)$ in Figure 10.7. On what range of x-values does it look like $P_4(x)$ approximates $\ln(x)$ well? (Eye-ball it and estimate to one decimal place.)

5. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** ________

Read Section 10.2, focusing on pages 546-547. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Taylor Series for** $\cos x$**,** $\sin x$**, and** e^x**.**
   (a) What is the Taylor series for $\cos x$ centered at zero?

   (b) What is the *general term* in the Taylor expansion for $\cos x$ about $x = 0$?

2. **Taylor Series in General.** Make sure to write the forumlas for Taylor series for $f(x)$ about $x = 0$ and for Taylor series for $f(x)$ about $x = a$ in your notebook.

3. **Convergence of Taylor Series.** Do not worry about the precise meaning of the word "convergence;" we will discuss this concept carefully at the end of the semester. An informal understanding of the notion is sufficient for our present purposes. Roughly speaking, if the Taylor polynomials P_0, P_1, P_2, $\ldots P_n$ $\ldots$ give better and better approximations for a function f in a certain range of x-values, then we say that the Taylor series *converges* to f on that interval of x-values.

   Reread the passage about the Taylor series for $\ln x$ about $x = 1$ on page 548, and look at Figure 10.10. True or false: $P_n(x)$ approximates $\ln(x)$ better and better as n gets larger, regardless of what x is. Explain.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 114, The Definite Integral: Area and Net Change (Sections 5.1, 5.2)

Name: ______________________________ **Section:** __________

Read Sections 5.1 and 5.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Distance and Velocity.** Review the notation on page 276, and try 5.1 #1, 13.

2. **The Definite Integral.** Try 5.2 #1, 2, 11.

3. What struck you in reading these sections? What is still unclear? What questions do you have?

Math 114, Numerical Methods for Definite Integrals (Section 7.5)

Name: ______________________ **Section:** __________

Read Section 7.5, focusing on the midpoint rule and the trapezoidal rule (pages 387-390). Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Midpoint and Trapezoid Rules.** Reread Examples 1 and 2.

   (a) For $\int_1^2 \frac{1}{x}\,dx$, list the vlaues of LEFT(2), RIGHT(2), MID(2), TRAP(2) in ascending order (from smallest to largest.)

   (b) Which of the four estimates are underestimates? Which are overestimates? Which estimate is the best?

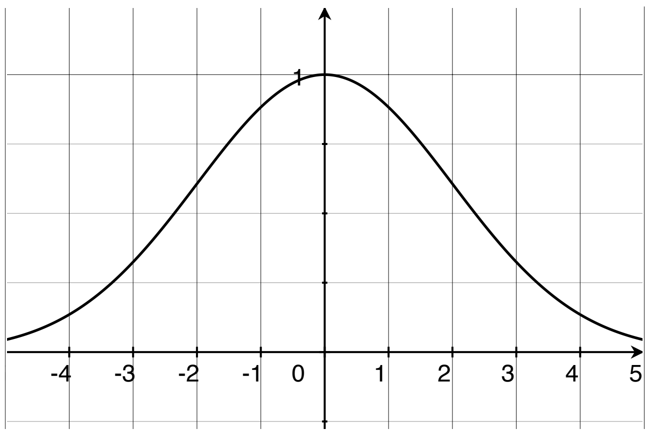
2. **Over- and Underestimates.** Reread pages 389-390.

   (a) If $f(x)$ is an *increasing* function, which of LEFT(n), RIGHT(n), MID(n), TRAP(n) for $\int_a^b f(x)dx$ is guaranteed to be an underestimate? an overestimate?

   (b) If $f(x)$ is *concave up*, which of LEFT(n), RIGHT(n), MID(n), TRAP(n) for $\int_a^b f(x)dx$ is guaranteed to be an underestimate? an overestimate?

3. Consider the integral $\int_0^2 e^{-x^2/8}\,dx$.

   (a) The graph of $e^{-x^2/8}$ is shown below. Given the shape of the graph, which of the four rules should give overestimates of the integral and which should give underestimates?

   (b) Compute LEFT(2), RIGHT(2), MID(2), and TRAP(2). (Round to six decimal places.)

4. What struck you in reading these sections? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Sections 5.3, 6.1, and 6.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Applications of the Fundamental Theorem.** Reread Example 3 in Section 5.3.

   (a) Suppose that one of your classmates says that neither car is ahead after one minute, because their graphs intersect at $t = 1$. Explain, in your own words, why this is incorrect.

   (b) Try Exercise 8, on page 294.

2. **Computing Values of an Antiderivative Using Definite Integrals** Reread pages 321-322, focusing on Example 4 and Example 5. Try 6.1 #1 and 2.

3. **Constructing Antiderivative Functions** Reread page 340 and Example 1.

   (a) Use the Construction Theorem for Antiderivatives to write down a formula for an antiderivative $F(x)$ of $f(x) = e^{-x^2}$. (Your formula should have an integral in it.)

   (b) This antiderivative function is important in the theory of probability. Use LEFT(2) or RIGHT(2) to estimate $F(1)$. Round your answer to six decimal places.

4. What struck you in reading these sections? What is still unclear? What questions do you have?

Math 114, Introduction to Differential Equations (Sections 6.3, and 11.1)

Name: ______________________________ **Section:** __________

Read Section 6.3, focusing on pages 332-333, and Section 11.1, focusing on pages 586-587. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Solving Simple Differential Equations with Antiderivatives.** Reread Examples 1 and 2 in Section 6.3. Imitate these examples to solve the following problems.

   (a) Find the general solution of the differential equation $\frac{dy}{dx} = 3x^2$.

   (b) Find the solution of the initial value problem $\frac{dy}{dx} = 3x^2$; $y(1) = 0$.

2. **How Fast Does a Person Learn?** Reread pages 586-587, about modeling learning with a differential equation, and answer the following questions.

   (a) Here, y stands for the percentage of the task already mastered. What does $\frac{dy}{dt}$ mean, in terms of learning the task? What happens to $\frac{dy}{dt}$ as y increases? Can you see this behavior in the graphs in Figure 11.1?

   (b) **Solving the Differential Equation Numerically** Suppose that, at the beginning of the employee's training, the employee has mastered 0% of the task. Use the numerical method that generates the data in Table 11.1 to find the y-value for $t = 6$.

(c) **A Formula for the Solution to the Differential Equation** According to the text, what is the general solution to the differential equation?

(d) **Finding the Aribitrary Constant** If we suppose that, at the beginning of the employee's training, the employee has mastered 0% of the task, we have the initial conditions: $y = 0$ when $t = 0$. What is the particular solution of the differential equation in this case?

Find the three particular solutions corresponding to the initial values: $y(0) = 50$, $y(0) = 100$, and $y(0) = 150$.

Which of the above solutions have meaningful practical interpretations, in terms of learning?

3. What struck you in reading these sections? What is still unclear? What questions do you have?

Name: ____ **Section:** __________

Read Section 11.2, focusing on pages 591-593. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Consider the differential equation $\frac{dy}{dx} = x - y + 1$.

   (a) Complete the table below by choosing x and y-values and finding $\frac{dy}{dx}$ at those values using the differential equation. For, example, with $x = 0$ and $y = 0$, we have $\frac{dy}{dx} = 0 - 0 + 1 = 1$.

   (b) Sketch a slope field for this differential equation, using your table of values by drawing a short line with the appropriate slope $\frac{dy}{dx}$ at that point. For example, at the point $(0, 0)$, draw a short line with slope 1, since $\frac{dy}{dx} = 1$ when $x = 0$ and $y = 0$.

   (c) Use your slope field to sketch three significantly different solution curves.

x	y	$\frac{dy}{dx}$
0	0	1
0	1	0
0	2	-1
0	3	
0	4	
0	-1	
0	-2	
0	-3	
-1	0	
-1	1	

2. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Separation of Variables and Modeling (Sections 11.4 and 11.5)

Name: ____ **Section:** __________

Read Section 11.4, focusing on pages 604-605, and Section 11.5, focusing on the population growth example on pages 609-610 and the heating/cooling examples on page 613 and page 615. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Separation of Variables.** Reread Examples 1 and 2 in Section 11.4.

   (a) What must be done to the differential equation before the integration step?

   (b) What must be done after the integration step to find the general solution?

   (c) Imitate Example 1 to solve the initial value problem: $\frac{dP}{dt} = .02P\,;\ P(0) = 100$.

   (d) Imitate Example 2 to solve the initial value problem: $\frac{dH}{dt} = -0.3(H - 20)\,;\ H(0) = 100$.

2. **Setting up a Differential Equation for Population Growth** Reread the introduction to 11.5 (bottom of page 609, top of page 610), about setting up a differential equation for unrestricted population growth.

   (a) In this model, the rate at which the population grows with respect to time is proportional to what quantity?

   (b) What does $\frac{dP}{dt}$ represent in the differential equation? What does 0.02 represent? P?

3. **A Differential Equation for Heating and Cooling** Reread the introduction to Newton's Law of Heating and Cooling (on page 613) and the discussion of equilibrium solutions (page 615).

   (a) In this model, the rate at which the coffee cools with respect to time is proportional to what quantity?

   (b) Suppose the coffee stands in a room where the temperature is 20°C. If $H(t)$ is the temperature, in °C, of the coffee at time t, in minutes, the differential equation modeling the cooling of the coffee is the differential equation

   $$\frac{dH}{dt} = -k(H - 20)$$

   In this equation what does $\frac{dH}{dt}$ represent? What does $(H - 20)$ represent?

   What is the equilibrium solution? Is it stable or unstable?

4. What struck you in reading these sections? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 7.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Simple Substitutions.** Reread Examples 1-4.

   (a) Imitate Example 1 (or Example 3) to find $\int 2xe^{x^2}\,dx$.

   (b) Imitate Example 2 (or Example 4) to find $\int x^4\cos(x^5)\,dx$.

2. **Less Simple Substitutions.** Reread Examples 5-7.

   (a) Find $\int \cos^5 x \sin x\,dx$. (Hint: Let $w = \cos x$.)

(b) Find $\int e^x \sqrt{1+e^x}\, dx$. (Hint: Let $w = 1 + e^x$.)

3. **Substitution with a Definite Integral** Reread Examples 9-11.

   (a) What are the two ways to use substitution to find definite integrals?

   (b) Find $\int_1^2 2xe^{x^2}\, dx$ using *both* ways.

4. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** ________

Read Section 7.2, focusing on Examples 1-5. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Straightforward IBP.** Reread Examples 1, 2, 4, and 5.

   (a) Imitate Examples 1 and 2 to find $\int x \sin x \, dx$.

   (b) Imitate Example 4 to find $\int x^2 \ln(x) \, dx$.

   (c) Imitate Example 5 to find $\int x^2 \sin(x) \, dx$. (Use IBP twice.)

2. **An antiderivative for** $\ln x$**.** Reread Example 3.

   (a) What are u and v' in this example?

   (b) What is the antiderivative of $\ln x$?

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 7.4, up to but not including the part about Trigonometric Substitutions, focusing on Examples 1-4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Denominator is Product of Distinct Linear Factors.** Reread Examples 1 and 2. We will imitate these examples to evaluate $\displaystyle\int \frac{2x+3}{x^2-4x-5}\,dx$.

   The denomniator factors as $(x+1)(x-5)$, and the partial fractions decomposition is of the form

   $$\frac{2x+3}{x^2-4x-5} = \frac{A}{x+1} + \frac{B}{x-5}$$

   (a) Clear denominators by multiplying both sides by $(x+1)(x-5)$.

   (b) Expand/foil the right hand side, and collect terms.

   You should get: $(A+B)x + (B-5A)$. This means that

   $$2x+3 = (A+B)x + (B-5A)$$

   Equating constant terms and coefficients of x gives: $2 = A+B$ and $3 = B-5A$. Use these two equations to solve for A and B.

   (c) Use the partial fractions decomposition to evaluate $\displaystyle\int \frac{2x+3}{x^2-4x-5}\,dx$.

2. **Denominator has a Repeated Linear Factor.** Reread Example 3. We will imitate this example to evaluate $\displaystyle\int \frac{x-9}{(x+5)(x-2)^2}\,dx$.

   Since there is a repeated linear factor, the partial fractions decomposition is of the form:

   $$\frac{x-9}{(x+5)(x-2)^2} = \frac{A}{x+5} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

   If we multiply both sides by $(x+5)(x-2)^2$ and then equate constant terms and coefficients of x, we get $A = -2/7$, $B = 2/7$, $C = -1$. Use this partial fractions decomposition to evaluate the integral.

3. **Denominator has a Quadratic that Cannot be Factored.** Reread Example 4. Suppose we wished to find the partial fractions decomposition for $\displaystyle\frac{x-9}{(x+5)(x^2-3x+9)}$.

   (a) Use the quadratic formula to show that $x^2 - 3x + 9$ cannot be factored.

   (b) What is the correct form of the partial fractions decomposition in this case? (You do not need to find the coefficients A, B, and C.)

4. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________ **Section:** ________

Read the part of Section 7.4 about Trigonometric Substitution, starting on page 380. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. In each of the examples, a trig identity ("the Pythagorean identity") is used to simplify the integrand after making a substitution. What is this identity? (See the paragraph before Example 6.)

2. For each of the examples listed below, state the trig substitution that is used. In particular, state both x and dx.

   (a) Example 6:

   (b) Example 7:

   (c) Example 8:

   (d) Example 10:

   (e) Example 11:

   (f) Example 12:

3. **Using a Triangle.** Reread Examples 8 and 9.

   (a) Draw and label the triangle used in Example 9.

   (b) What is $\cos\theta$, in terms of x?

   (c) What would $\tan\theta$ be, in terms of x, if we needed to find it?

4. For each expression below, state an appropriate trig substitution that could be used to simplify. State both x and dx.

   (a) $\sqrt{a^2 - x^2}$

   (b) $a^2 + x^2$

   (c) $\sqrt{a^2 - (x - h)^2}$

   (d) $a^2 + (x - h)^2$

5. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 1.8, focusing on pages 57-58, 61-63. Do not worry about the rigorous ϵ, δ definition of the limit on page 59. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Local Behavior** For each item below, sketch a graph of a function with the desired properties. (Examples in the section will work for some of these, but you could also create your own examples.)

   (a) $f(0)$ is undefined; $\lim_{x\to 0} f(x) = 1$

   (b) $f(0)$ is undefined; $\lim_{x\to 0^-} f(x) = -\infty$; $\lim_{x\to 0^+} f(x) = \infty$

   (c) $f(0)$ is undefined; $\lim_{x\to 0} f(x)$ does not exist

   (d) $f(0)$ is undefined; $\lim_{x\to 0^-} f(x) = -1$; $\lim_{x\to 0^+} f(x) = 1$

2. **End Behavior.** Use what you know about the graphs of the functions to investigate the limits at infinity.

   (a) $\lim_{x\to\infty} e^x$, $\lim_{x\to-\infty} e^x$

   (b) $\lim_{x\to\infty} x^3$, $\lim_{x\to-\infty} x^3$

   (c) $\lim_{x\to\infty} \ln|x|$, $\lim_{x\to-\infty} \ln|x|$

3. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, L'Hopital's Rule (Section 4.7)

Name: ______________________________ **Section:** __________

Read Section 4.7, focusing on pages 242-245. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **l'Hopital's Rule for Quotients.** Reread Examples 1, 2, and 3.

   (a) Imitate Example 1 to evaluate $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{x}$.

   (b) Imitate Example 2 to evaluate $\lim_{x\to \infty} \dfrac{e^x - e^{-x}}{x}$.

2. **Dominance.** Consider the functions $f(x) = e^x - e^{-x}$ and $g(x) = x$.

   (a) Evaluate $\lim_{x\to\infty} \dfrac{g(x)}{f(x)}$.

   (b) Given your answer to 2(a) and 1(b), which function dominates the other: $f(x)$ or $g(x)$?

3. **Rewriting Indeterminate Forms.** Reread Example 6.

   (a) What is the form of the limit in this example?

   (b) How do we rewrite the limit so as to be able to use l'Hopital's rule?

4. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 7.6, focusing on pages 395-397 and 398-399. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Improper Integrals.** Reread the first paragraph of the section.
   (a) What two things did we assume in our original discussion of the definite integral $\int_a^b f(x)dx$?

   (b) What are the two kinds of improper integrals?

2. **When the Limit of Integration is Infinite** Reread the discussion of the improper integral $\int_1^\infty \frac{1}{x^2}\,dx$ on pages 395-396 as well as Example 1, about $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$, and the paragraph following it.
   (a) Which of these improper integrals converges? Which diverges?

   (b) What is the difference between the functions $1/x^2$ and $1/\sqrt{x}$ that makes the area under the graph of $1/x^2$ aproach 1 as $x \to \infty$, whereas the area under $1/\sqrt{x}$ grows very large?

3. Reread Example 3, about the family of improper integrals $\int_1^\infty \frac{1}{x^p}\,dx$.
   (a) Does $\int_1^\infty \frac{1}{x}\,dx$ converge? If so, what is its value?

   (b) Does $\int_1^\infty \frac{1}{x^3}\,dx$ converge? If so, what is its value?

4. **When the Integrand Becomes Infinite.** Reread pages 398-400. All but one of the following integrals are improper. Which integral is not improper? For each integral that *is* improper, identify the "trouble spot."

   (a) $\int_0^1 \frac{1}{x^2}\,dx$

   (b) $\int_1^2 \frac{1}{x^2}\,dx$

   (c) $\int_1^2 \frac{1}{(x-2)^2}\,dx$

   (d) $\int_0^2 \frac{1}{(x-1)^2}\,dx$

5. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Parametric Equations (Section 4.8-I)

Name: ______________________________ **Section:** __________

Read the first part of Section 4.8, up to and including Example 8 on page 253. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Circular and Linear Motion.** Reread Examples 1, 3, and 5.

   (a) Describe the motion of the particle whose x and y coordinates at time t are given by the equations $x = \cos(t/2)$ and $y = \sin(t/2)$.

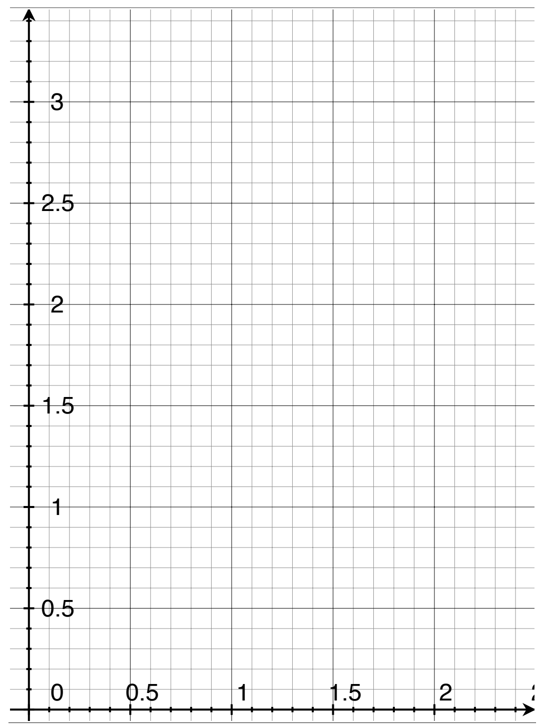
   (b) Write parametric equations for a particle moving with constant speed along a straight line from $(1, 2)$ to $(-1, 3)$. What is the slope of this line?

2. The motion of a particle in the xy-plane is described in terms of time, t, by

$$x = \ln(t) \quad y = \sqrt{t} \quad t \geq 1$$

(a) Make a table of x and y coordinates for the curve along which the particle moves. (Round to one decimal place.) Sketch a graph of the curve by plotting the points in your table. Indicate with an arrow the direction in which the curve is traced as t increases.

t	x	y
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		



(b) Find the velocity in the x-direction, the velocity in the y-direction, and the speed when $t = 1$.

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read the second part of Section 4.8, **as well as the hand-out on finding area under a parametric curve**. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Slope and Concavity.** Reread Example 9 and the discussion of slope and concavity, p 255-256.

   Consider the parametric curve in Example 9: $x = t^3$, $y = 2t$.

   (a) According to Example 9, what are $\frac{dy}{dt}$ and $\frac{dx}{dt}$?

   (b) According to Example 9, what are the parametric equations for the tangent line to the curve at the point $(1, 2)$? What is the slope of this line? (You may use the formula on page 251.)

   (c) Using the equation for the slope of a parametric curve on page 255, what is $\frac{dy}{dx}$ as a function of t? What is the slope of the parametric curve when $t = 1$?

   (d) Find the point on the parametric curve corresponding to $t = 1$. Use this point and the slope you found in (d) to find an equation for the tangent line to the curve at when $t = 1$.

   (e) Let $m(t)$ be the slope of the parametric curve as a function of t (the one you found in (d)). What is $m'(t)$? To find $\frac{d^2y}{dx^2}$, divide $m'(t)$ by $x'(t)$. Is the curve concave up or concave down when $t = 1$?

2. **Area.** Consider the parametric curve in Example 9: $x = t^3$, $y = 2t$. We will find the area between the curve and the x-axis from $x = 0$ to $x = 8$.

   (a) Notice that when t increases, $x = t^3$ also increases, so the curve is traversed from left to right. What are the t-values corresponding to $x = 0$ and $x = 8$? Call them α and β, respectively.

   (b) Check that the curve is above the x-axis ($y \geq 0$) when t is between α and β.

   (c) Using the fact that $dx = x'(t)dt$, find dx.

   (d) Find the area between the curve and the x-axis from $x = 0$ and $x = 8$ using the area formula:

$$A = \int_{x(\alpha)}^{x(\beta)} y\,dx = \int_{\alpha}^{\beta} y(t)\,x'(t)\,dt$$

3. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Area Between Two Curves; Average Value (Section 5.4)

Name: ______________________ **Section:** ________

Read "Area Between Curves" and "The Definite Integral as an Average" in Section 5.4, pages 301-302 and pages 304-305. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Area Between Two Curves.** Reread Example 3. Now consider the region below the curve $y = 5x - x^2$ and above the curve $y = x$.

   (a) Where do the two curves cross? (Solve the equation $x = 5x - x^2$ for x.)

   (b) Sketch the region.

   (c) Find the area of this region by subtracting one definite integral from another.

2. **Average Value**

   (a) Reread Example 6. If $V(t)$ represents the value, in dollars, of a Tiffany lamp t years after 1975, write an expression (involving an integral) that represents the average value of the lamp over the period 1975-2010.

(b) Find the average value of the function $f(x) = 5x - x^2$ over the interval $[0, 5]$. Sketch a graph of the function on this interval. Does your answer make sense?

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 8.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Area: Vertical and Horizontal Slices.** Reread Example 1. Now consider the triangle in Exercises 1 and 3, page 419.

   (a) Use the formula $A = \frac{1}{2}bh$ to find the area of this triangle.

   (b) Find the area of the triangle using vertical slices, as in Exercise 1. Note that the (very small) width of each slice is Δx and the height of each slice is $h(x) = 2x$.

   (c) Find the area of the triangle using horizontal slices, as in Exercise 3. Note that the (very small) height of each slice is Δy. Use similar triangles to find the width w of each slice in terms of y. You will integrate $w(y)$ with respect to y, with y ranging from $y = 0$ to $y = 6$.

2. **Volume.** Reread page 416, which shows how to find the volume of a cone.

   (a) In Example 3, we slice the cone into horizontal slices. Why not slice the cone vertically, as shown in Figure 8.5? (See the paragraph before the example.)

   (b) When we slice the cone horizontally, each slice is approximately a disk. We will find the volume of each disk and add up the volumes of each disk to approximate the volume of the cone.

   What is the thickness of each disk?

   What is the radius of each disk?

   What is volume of each disk?

   (c) What is the Riemann sum that approximates the volume of the cone? What is the integral that expresses the exact volume of the cone? (Don't evaluate the integral; just write it down.)

3. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Applications to Geometry: Volumes by Slicing (Section 8.2-I)

Name: ______________________ **Section:** __________

Read Section 8.2, pages 422-425, up to but not including the part about arc length. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Rotating Around the x-axis.** Reread Example 1.

   (a) Sketch the planar region described in Example 1. Then, in a separate drawing, sketch the solid obtained by revolving this region around the x-axis.

   (b) We slice the solid perpendicularly to the axis of rotation. What is the shape of a typical slice? (A circular disk or a circular washer?) What is the volume of a typical slice?

   (c) Write (but do not evaluate) the integral representing the volume of the solid.

   (d) Now try Exercise 1 on page 427. Include a sketch of the solid.

2. **Rotating Around Another Horizontal Line.** Reread Example 3.

   (a) Sketch the planar region described in Example 3 and the line $y = 3$. Then, in a separate drawing, sketch the solid obtained by revolving this region around the line $y = 3$.

   (b) We slice the solid perpendicularly to the axis of rotation. What is the shape of a typical slice? (A circular disk or a circular washer?) What is the volume of a typical slice?

   (c) Write (but do not evaluate) the integral representing the volume of the solid.

3. **Another Way to Construct Solids.** Reread Example 4

   (a) What shape is the typical slice in this example? What is the volume of a typical slice?

   (b) Write (but do not evaluate) the integral representing the volume of the solid.

4. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Applications to Geometry: Arc Length (Section 8.2-II)

Name: ______________________________ **Section:** __________

Read Section 8.2, pages 425-427. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 5.

   (a) Use the `Plot` command in *Mathematica* to plot the curve.

   (b) What is the integral for the arc length in this example?

   (c) Use the `NIntegrate` command in *Mathematica* to estimate the arc length, and write your estimate below.

   (d) Now consider the curve $y = x^2$ from $x = 0$ to $x = 3$. Use the `Plot` command in *Mathematica* to plot the curve. Sketch the curve below.

   (e) What is the integral for the arc length of $y = x^2$ from $x = 0$ to $x = 3$?

   (f) Use the `NIntegrate` command in *Mathematica* to estimate the arc length, and write your estimate below. Does this estimate seem reasonable? Explain.

2. Reread Example 6.

   (a) Use the `ParametricPlot` command in *Mathematica* to plot the curve.

   (b) What is the integral for the arc length in this example? Use the `NIntegrate` command in *Mathematica* to estimate the arc length.

   (c) Now consider the parametric curve $x = 20 \sin t$, $y = t^2$, $0 \leq t \leq 2\pi$. Use the `ParametricPlot` command in *Mathematica* to plot the curve. Sketch the curve below.

   (d) What is the integral for the arc length of $x = 20 \sin t$, $y = t^2$, $0 \leq t \leq 2\pi$?

   (e) Use the `NIntegrate` command in *Mathematica* to estimate the arc length.

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ____________________ **Section:** ________

Read Section 9.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Sequences.** Reread Examples 1 and 3. Give the first six terms for the following sequences:

   (a) $s_n = 2^n + 1$, $n \geq 1$.

   (b) $s_n = s_{n-1} + s_{n-2}$ for $n > 2$ and $s_1 = 1$, $s_2 = 1$

2. **Convergence of Sequences** What are the "two facts" that we can use to calculate the limit of a sequence, in addition to what we know about the limits of functions?

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 9.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Sum of a Finite Geometric Series.** Reread the paragraph on the sum of a finite geometric series on page 500. Now consider the finite geometric series $5 + 5/3 + 5/9 + 5/27$.

   (a) What is the first term of this geometric series?

   $a \ =$

   (b) What is the common ratio for this geometric series?

   $x \ =$

   (c) How many terms are there in this geometric series?

   $n \ =$

   (d) Calculate the sum of this finite geometric series, using the formula on page 500.

2. **Sum of an Infinite Geometric Series.** Reread the passage about the sum of an infinite geometric series on pages 500 to 501. Consider the infinite geometric series: $5 + 5/3 + 5/9 + 5/27 + \ldots$

   (a) Does the series converge or diverge? How can you tell, without taking a limit?

   (b) Give a formula for S_n, the nth partial sum of this series.

   (c) What is the sum of the infinite series?

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 9.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **The Limit of Partial Sums.** Reread Example 1.

   (a) What is a_n in this example? S_n?

   $a_n \quad =$ $S_n \quad =$

   (b) What is $\lim_{n\to\infty} a_n$? $\lim_{n\to\infty} S_n$?

   $\lim_{n\to\infty} a_n \quad =$ $\lim_{n\to\infty} S_n \quad =$

   (c) What is the sum of the series?

   $S \quad =$

2. **Convergence Properties of Series.** Reread Theorem 9.2.

   (a) Use Theorem 9.2.1 to explain why $\sum_{n=1}^{\infty}\left((\frac{1}{2})^{n-1} + (\frac{1}{3})^{n-1}\right)$ is convergent and to find the sum.

   Hint: Notice $\sum_{n=1}^{\infty}(\frac{1}{2})^{n-1}$ is geometric; the first term is $a = 1$ and the common ratio $x = \frac{1}{2}$.

   (b) Use Theorem 9.2.3 to explain why $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges.

   (c) Use (b) and Theorem 9.2.4 to explain why $\sum_{n=1}^{\infty} \frac{5n}{n+1}$ diverges.

3. **An Important Family of Examples.** Reread Examples 3, 4, and 5.

   (a) What is the name of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$? Does it converge or diverge?

   (b) State whether the following series converge or diverge. (Use Theorem 9.2 and Example 5.)

$$\sum_{n=1}^{\infty} \frac{1}{n^6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

$$\sum_{n=1}^{\infty} \frac{5}{n^6}$$

$$\sum_{n=1}^{\infty} \left(\frac{2}{n^6} + \frac{3}{n^2}\right)$$

$$\sum_{n=1}^{\infty} \frac{3}{n^{1/2}}$$

4. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________ **Section:** ________

Read the part of Section 9.4 about the Ratio Test, pages 515-516. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 6.

   (a) What is a_n in this example? What is $|a_{n+1}|/|a_n|$?

   $$a_n \quad = \qquad\qquad\qquad\qquad \frac{|a_{n+1}|}{|a_n|} \quad =$$

   (b) What is $\lim\limits_{n\to\infty} a_n$? $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|}$?

   $$\lim_{n\to\infty} a_n \quad = \qquad\qquad\qquad\qquad \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} \quad =$$

   (c) What does the Ratio Test allow use to conclude about this series?

2. Reread Example 7. Consider the first series in this example, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

   (a) What is a_n for this series? What is $|a_{n+1}|/|a_n|$?

   $$a_n \quad = \qquad\qquad\qquad\qquad \frac{|a_{n+1}|}{|a_n|} \quad =$$

   (b) What is $\lim\limits_{n\to\infty} a_n$? $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|}$?

   $$\lim_{n\to\infty} a_n \quad = \qquad\qquad\qquad\qquad \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} \quad =$$

   (c) What does the Ratio Test allow use to conclude about this series? What do we know about this series from Section 9.3?

3. Now consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$.

   (a) What is $|a_{n+1}|/|a_n|$?

   Hints: Here are a few things to keep in mind: (1) dividing by a fraction is the same as multiplying by the reciprocal, (2) for any number a, we know that $a^{n+1}/a^n = a$, and (3) as in Example 6, the ratio of successive factorials cancels nicely: $n!/(n+1)! = 1/(n+1)$.

   (b) What is $\displaystyle\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$?

   (c) What does the Ratio Test allow use to conclude about this series?

4. What struck you in reading this section? What is still unclear? What questions do you have?

Math 114, Power Series and Radius of Convergence (Section 9.5)

Name: ______________________________ **Section:** __________

Read Section 9.5, focusing on the ideas of power series and radius of convergence. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Geometric Power Series.** Reread Example 2.

   (a) Determine whether the given power series converges for $x = 1$.

   (b) Notice that this is a geometric series with first term 1 and common ratio $\frac{x}{2}$. Use what you know about the convergence of geometric series to determine for what values of x the series converges.

   (c) Use the formula for the sum of an infinite geometric series to write this power series as a rational function.

2. **Non-geometric Power Series.** Reread Examples 3 and 4. Notice that these power series are *not* geometric, so we need to use the Ratio Test to determine for what values of x the series converges.

   (a) In Example 3, what is a_n? What is $|a_{n+1}|/|a_n|$?

   (b) Continuing with Example 3, what is $\lim_{n\to\infty} |a_{n+1}|/|a_n|$?

(c) Continuing with Example 3, for what values of x is $\lim_{n\to\infty} |a_{n+1}|/|a_n| < 1$? (In other words, for what values of x does the series converge?)

(d) Now look at Example 4. In this example, what is a_n? What is $|a_{n+1}|/|a_n|$?

(e) Continuing with Example 4, what is $\lim_{n\to\infty} |a_{n+1}|/|a_n|$?

(f) Continuing with Example 4, for what values of x is $\lim_{n\to\infty} |a_{n+1}|/|a_n| < 1$? (In other words, for what values of x does the series converge?)

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________ **Section:** ________

Read Section 10.2 and the beginning up Section 10.3, up through Example 1 on page 553. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Binomial Series Expansions.** Consider the function $f(x) = \frac{1}{1+x}$.

   (a) Repeatedly differentiate $f(x)$: find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$. Simplify along the way!

   (b) Evaluate $f(x)$ and its derivatives at $x = 0$: find $f(0)$, $f'(0)$, $\ldots$, $f^{(4)}(0)$.

   (c) Find the first five Taylor coefficients for $f(x)$ centered at $x = 0$, using the fact that $C_n = \frac{f^{(n)}(0)}{n!}$.

   (d) Write out the beginning of the Taylor series for $f(x)$, centered at $x = 0$. Include the first five terms.

(e) Compare your answer to the answer you would obtain by using the formula for the binomial series expansion in the bottom of page 549, as in Example 2.

2. **New Series by Substitution.** Reread the beginning of Section 10.3, including Example 1. Using the Taylor series for $f(x) = \frac{1}{1+x}$, find a Taylor series for $g(x) = \frac{1}{1+2x}$. For what values of x does this series converge?

3. What struck you in reading this section? What is still unclear? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 10.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. **Differentiating a Taylor Series.** Reread Example 2. Continue this example to find a Taylor series about $x = 0$ for $\dfrac{2}{(1-x)^3}$ from the series for $\dfrac{1}{(1-x)^2}$, as outlined below.

   (a) What is the derivative of $\dfrac{1}{(1-x)^2}$? (Rewrite the function as $(1-x)^{-2}$, then use the power rule and the chain rule.)

   (b) What is the Taylor series about $x = 0$ for $\dfrac{1}{(1-x)^2}$? (This is given in Example 2.)

   (c) Differentiate the Taylor series about $x = 0$ for $\dfrac{1}{(1-x)^2}$ term by term.

   (d) What is the Taylor series about $x = 0$ for $\dfrac{2}{(1-x)^3}$? (This should be your answer in (c).)

2. **Integrating a Taylor series.** Reread Example 3. Imitate the ideas in this example to find a Taylor series about $x = 0$ for $\ln(1 - x)$ from the series for $\dfrac{1}{1-x}$, as outlined below.

   (a) Find $\displaystyle\int \frac{1}{1-x}\,dx$.

   (b) What is the Taylor series about $x = 0$ for $\dfrac{1}{1-x}$? (Look back at Example 1.)

   (c) Integrate the Taylor series about $x = 0$ for $\dfrac{1}{1-x}$ term by term. (Don't forget the $+C$!)

   (d) What is the Taylor series about $x = 0$ for $\ln(1 - x)$? (Use your answer from (c).)

3. What struck you in reading this section? What is still unclear? What questions do you have?