Math 114, Quiz May 13, 2016

Name: _____ **Section:** _____

Instructions: This quiz has five problems, each of which is worth ten points. Make sure to show all your work and make your final answer clear. Include labels and units when appropriate. No notes, books, or calculators are permitted during the quiz.

1. (*10 points*) Fill in the blanks.

   When $|x| < 1$, the geometric series $\sum_{n=0}^{\infty} ax^n$ converges to _____ .

   For p _____ , the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges.

   Suppose that $\{S_n\}$ is the sequence of partial sums for the series $\sum_{n=1}^{\infty} a_n$ and $\lim_{n\to\infty} S_n = 10$. Then the series $\sum_{n=1}^{\infty} a_n$ _____ . (*diverges* or *converges to ...*)

   The harmonic series, $\sum_{n=1}^{\infty}$ _____ , is an example of a _____ (*convergent* or *divergent*) series.

   Suppose the sum of the series $\sum_{n=0}^{\infty} a_n$ is 3, and let S_n denote the n^{th} partial sum.

   Then $\lim_{n\to\infty} a_n$ = _____ and $\lim_{n\to\infty} S_n$ = _____ .

   Suppose the Taylor series for a function $g(x)$ centered at $x = 0$ is: $2 - x + 2x^2 - x^3 + 2x^4 - x^5 + \ldots$

   Then $g(0)$ = _____ , $g'(0)$ = _____ , and $g''(0)$ = _____ .

2. (*10 points*) Let $a_n = \dfrac{1}{n} - \dfrac{1}{n+1}$.

   (a) Find the limit of the sequence $\{a_n\}$.

   (b) Consider the series $\displaystyle\sum_{n=1}^{\infty} a_n$.

      i. List the first three partial sums s_1, s_2, s_3 of the series.

      ii. Find a closed formula for s_n.

      iii. Find the sum of the series.

3. (*10 points*) Dr. LaValle drinks two cups of coffee per day, but she would like to gradually wean herself off of coffee, so she decides to reduce her coffee intake by 10% each day, starting tomorrow.

   (a) How much coffee does Dr. LaValle drink today? tomorrow? in n days?

   (b) How much coffee does Dr. LaValle drink, in total, over the first three days of her coffee intake reduction scheme?

   (c) How much coffee does Dr. LaValle drink, in total, over the first n days of her coffee intake reduction scheme?

   (d) What is the maximum amount of coffee that Dr. LaValle will drink over the course of the rest of her life, starting today?

4. (*10 points*) Find the center and radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{5^n\, n^2}$.

5. (*10 points*) Estimate $\int_0^1 \cos(x^2)\,dx$ using a Taylor polynomial with three nonzero terms.

Scratch paper. (*If you want your work on this page to be graded, make sure to label your work according to the problem you're solving, and make sure to write a note next to the original problem.*)