

This exam covers:

- Techniques of Integration: substitution, ibp, partial fractions, trig substitution (7.1, 7.2, 7.4)
- Limits and Improper Integrals: the limit of the values of a function, infinite limits, limits at infinity, indeterminate forms, L'Hopital's rule, convergent and divergent integrals (1.8, 4.7, 7.6)
- Parametric Equations: parametric equations of motion, velocities in x and y directions, speed, parametric curves: slope, concavity, area (4.8 and supplementary notes)

Textbook exercises for review:

- Ch 1 Review: 44, 45, 46
- Ch 4 Review: 7*, 8*, 9*, 37, 38, 82, 83, 84, 85
*Find the limits only.
- Ch 7 Review: 3-69, 72, 77-78, 88, 112, 115, 119, 120, 122, 125, 126a, 128-134, 140-151, 154, 181, 189

Additional exercises for review:

1. Consider the parametric equations $x = \sqrt{t}$, $y = 1/t$.
 - (a) Find an equation for the tangent line to the curve at the point $(1, 1)$.
 - (b) Determine whether the curve is concave up or concave down at the point $(1, 1)$.
 - (c) Find the area under the curve from $x = 1$ to $x = 4$.
2. Consider the parametric equations $x = 2t^3 + 1$, $y = t^2$.
 - (a) Find the area enclosed by the curve and the x -axis from $x = 1$ to $x = 3$ by setting up and evaluating an integral in terms of t .
 - (b) Check your answer by eliminating the parameter to obtain a Cartesian equation for the curve and integrating with respect to x .
3. Consider the parametric equations $x = 2t$, $y = 3 - 3t^2$.
 - (a) Find the area between the curve and the x -axis by setting up and evaluating an integral in terms of t .
 - (b) Check your answer by eliminating the parameter to obtain a Cartesian equation for the curve and integrating with respect to x .
4. The motion of a reflector on a bicycle wheel of radius 1 is given by $x = t - \sin t$, $y = 1 - \cos t$. The path traced out by the reflector is a curve called a **cycloid**.
 - (a) Find the velocity of the reflector in the x -direction, the velocity of the reflector in the y -direction, and the speed of the reflector at arbitrary time t .
 - (b) Are there any moments when the reflector is momentarily paused? When its motion is purely up and down? Purely left and right?
 - (c) Find the horizontal and vertical tangents of the cycloid.
 - (d) Where is the cycloid concave up? Concave down?
 - (e) Sketch the cycloid by plotting x and y coordinates for $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \dots$
 - (f) Find the area under one arch of the cycloid.