Math 132, Prep 6.2 (Volumes: Slices)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. Solids of rotation: discs, washers; horizontal, vertical axes of rotation
2. Other solids whose cross-sectional area can be described using geometry

One way to approximate the volume of a solid object is to "slice" the object into thin pieces, approximate the volumes of the pieces, and then add these up. Using thinner and thinner slices will yield better and better approximations, and the limiting value of the approximations is the exact volume of the solid.

If the cross-sectional area of a solid at x is $A(x)$, then the infinitesimal area of the slice at x is $dV = A(x)dx$. Integrating gives the total volume of the solid:

$$V = \int dV = \int_a^b A(x)\,dx$$

1. Solids of Revolution

When the solid is obtained by rotating a region in the plane about some axis, then the cross-sections will always be circles or annuli (rings). It's worth remembering that

$$(\text{area of circle of radius } r) = \pi r^2$$
$$(\text{area of annulus of outer radius } R \text{ and inner radius } r) = \pi(R^2 - r^2)$$

Exercises.

1. In this problem we will find the volume of the solid obtained by rotating the region bounded by

$$y = e^x \qquad y = 0 \qquad x = 0 \qquad x = 1$$

around the x-axis. (See Examples 2 and 4 in the textbook.)

(a) Sketch the region bounded by the given curves.

(b) Sketch the solid obtained by rotating the region around the x-axis.

(c) We imagine slicing the solid into thin pieces, slicing in a plane perpendicular to the axis of rotation, in this case, the x-axis. What is the shape of the typical slice? A disc? A washer?

(d) What is the thickness of the typical slice? Is it a dx or a dy?

(e) What is the volume (dV) of the typical slice?

(f) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

2. We will find the volume of the region bounded by the curves

$$y = \ln(x) \qquad y = 1 \qquad y = 2 \qquad x = 0$$

around the y-axis. (See Example 3 in the textbook.)

(a) Sketch the region bounded by the given curves. Sketch the axis of revolution as well. In a separate picture, sketch the solid obtained by rotating the region around the axis of revolution.

(b) Draw a typical slice of the solid. What is the volume (dV) of a typical slice?

(c) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

3. We will find the volume of the region bounded by the curves

$$y = \frac{1}{x} \qquad y = 0 \qquad x = 1 \qquad x = 3$$

around the axis $y = -1$. (See Example 5 in the textbook.)

(a) Sketch the region bounded by the given curves. Sketch the axis of revolution as well. In a separate picture, sketch the solid obtained by rotating the region around the axis of revolution.

(b) Draw a typical slice of the solid. What is the volume (dV) of a typical slice?

(c) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

4. We will find the volume of the region bounded by the curves

$$y = x \qquad y = \sqrt{x}$$

around the axis $x = 2$. (See Example 6 in the textbook.)

(a) Sketch the region bounded by the given curves. Sketch the axis of revolution as well. In a separate picture, sketch the solid obtained by rotating the region around the axis of revolution.

(b) Draw a typical slice of the solid. What is the volume (dV) of a typical slice?

(c) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

5. **Challenge.** Consider the region bounded below by the x-axis, bounded above by the curve $y = \frac{1}{x}$, and bounded on the left by $x = 1$. We know that this region has infinite area. Show that, if we rotate this region around the x-axis, the resulting solid (called "Gabriel's Horn") has a finite volume.

2. Other Solids

Recall that the volume of a solid can be found by integrating its cross-sectional area. Some solids have a cross-sectional area that can be described with a formula, even though they are not solids of revolution. See Examples 7-9.

Exercises.

6. Find the volume of the described solid S. In each case, the base of S is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis.

   (a) Cross-sections perpendicular to the y-axis are squares.

(b) Cross-sections perpendicular to the x-axis are isosceles triangles with height equal to the base.