Name: _____ **Section:** _____

Names of collaborators: _____

Main Points:

1. Using cyllindrical shells to approximate the volume of a solid of revolution
2. Horizontal, vertical axes of rotation

The method of cylindrical shells is useful for computing volumes when the slicing method is difficult. We can approximate the volume of a solid of revolution using thin concentric cylindrical shells, which emanate outward from the axis of rotation.

The volume of a cylindrical shell is its surface area times its thickness (dx or dy depending on whether the axis of rotation is vertical or horizontal.) For example, if the axis of rotation is vertical,

$$dV = (\text{surface area}) \times (\text{thickness}) = (\text{circumference} \times \text{height}) \times (\text{thickness}) = (2\pi rh)(dx)$$

Exercises.

1. In this problem we will find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad y = 0 \quad x = 1$$

around the y-axis. (See Examples 1 and 2 in the textbook.)

(a) Sketch the region bounded by the given curves. In a separate picture, sketch the solid obtained by rotating the region around the y-axis.

(b) Sketch a typical shell. In this case, since the axis of rotation is vertical, the thickness of a shell is dx. Find formulas for the radius and height of the shell, in terms of x.

(c) What is the volume (dV) of the typical shell?

(d) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

2. We will find the volume of the solid obtained by rotating the region bounded by

$$x + y = 3 \qquad x = -y^2 + 2y + 3$$

around the x-axis. (See Example 3 in the textbook.)

(a) Sketch the region bounded by the given curves. In a separate picture, sketch the solid obtained by rotating the region around the x-axis.

(b) Sketch a typical shell. In this case, since the axis of rotation is horizontal, the thickness is dy. Find formulas for the radius and height in terms of y. What is the volume (dV)?

(c) Write an integral for the volume of the solid, and evaluate the integral to find the volume.

3. We will find an integral for the volume of the solid obtained by rotating the region bounded by

$$y = x \qquad y = 4x - x^2$$

around the axis $x = 7$. (See Example 4 in the textbook.)

(a) Sketch the region bounded by the given curves. In a separate picture, sketch the solid obtained by rotating the region around the axis $x = 7$.

(b) Sketch a typical shell. What is the volume (dV)?

(c) Write an integral for the volume of the solid. You do not need to evaluate the integral.

4. We will find an integral for the volume of the solid obtained by rotating the region bounded by

$$y = x^3 \qquad x = 1 \qquad y = 0$$

around the axis $y = 1$. (See Example 4 in the textbook.)

(a) Sketch the region bounded by the given curves. In a separate picture, sketch the solid obtained by rotating the region around the axis $y = 1$.

(b) Sketch a typical shell. What is the volume (dV)?

(c) Write an integral for the volume of the solid. You do not need to evaluate the integral.