Math 132, Prep 8.2 (Area of a Surface of Revolution)

Name: ______________________ **Section:** __________

Names of collaborators: ______________________

Main Points:

1. Revolving an arc of a curve around a horizontal line
2. Revolving an arc of a curve around a vertical line

Given an arc of a curve, we may revolve it around an axis; doing so results in a surface of revolution. To find the area of this surface, we divide it into pieces, cutting perpendicular to the axis of rotation. Each piece can be approximated by a band of a cone, and the area of a piece is approximated by the area of a band of a cone:

$$\Delta S \approx 2\pi r l \ ,$$

where r is the average radius of the band on the cone and l is the "slant height" of the band on the cone. (See Figure 3 in the textbook.) Using the arc length differential, we can rewrite this:

$$dS = 2\pi r \, ds \ .$$

To find the total area of the surface, we integrate.

Exercise.

1. Consider the part of the curve $y = x^3$ where $0 \leq x \leq 2$. We rotate this curve around the x-axis to create a surface of revolution.

   (a) Draw the piece of the curve and the surface of revolution (separately).

   (b) We cut the surface perpendicular to the axis of rotation. Draw a "typical" piece, and find its radius (in terms of x).

(c) Find ds and dS.

(d) Set up an integral for the area of the surface, and evaluate it.

2. Find the exact area of the surface obtained by rotating the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}\,; \qquad \tfrac{1}{2} \le x \le 1$$

about the x-axis.

3. Find the exact area of the surface obtained by rotating the curve

$$x^{2/3} + y^{2/3} = 1 \, ; \quad 0 \le y \le 1$$

about the y-axis.

Hint. After setting up the integral, you will have an improper integral with respect to y, because the integrand has a discontinuity at zero.

4. If the infinite curve e^{-x} with $x \geq 0$ is rotated around the x-axis, find the area of the resulting surface.

   Hint. After setting up the integral, do a u-substitution with $y = e^{-x}$. You may use Integral Formula 21 from the text, which says:

$$\int \sqrt{1+u^2}\, du \;=\; \tfrac{1}{2}u\sqrt{1+u^2} \;+\; \tfrac{1}{2}\ln(u+\sqrt{1+u^2})\,.$$