Name: ______________________________

Names of collaborators: ______________________________

Main Points:

1. basic definitions: series, partial sums, sum of series, convergence/divergence
2. geometric series, harmonic series
3. test for divergence

1. Series

Consider a sequence $\{a_1, a_2, a_3, \ldots\}$. A **series** is what we use to try to determine the accumulation of these values:

$$\sum_{n=1}^{\infty} a_n \quad = \quad a_1 + a_2 + a_3 + \ldots$$

Of course, there is no guarantee that this actually represents a finite number. To make this more precise we need to look at a new sequence, the sequence of partial sums.

Read the beginning of Section 11.2, including Definition 1 in the red box, and stopping before Example 1.

Exercises.

1. Consider the series
$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$$

   (a) Find a formula for the n^{th} term of the series. (Start your index, n, with $n = 1$.)

   $a_n \quad =$

   (b) Write the series in sigma notation.

   (c) Calculate the first several partial sums:

   $s_1 \quad = \quad \frac{1}{3} \quad =$

   $s_2 \quad = \quad \frac{1}{3} + \frac{1}{9} \quad =$

   $s_3 \quad = \quad \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \quad =$

   $s_4 \quad = \quad \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \quad =$

2. Geometric Series and Harmonic Series

Two very important examples are geometric series and harmonic series.

Exercises.

2. Read about the Sum of a Geometric Series.

   (a) Consider a geometric series $a + ar + ar^2 + \ldots \;=\; \sum_{n=1}^{\infty} ar^{n-1}$, $(a \neq 0)$. Fill in the blanks.

   If $r \neq 1$, the nth partial sum of the geometric series is $s_n \;=\;$ __________________ .

   The geometric series is convergent if __________________ , and (in this case) its sum is:

   $$\sum_{n=1}^{\infty} ar^{n-1} \;=\; \underline{\hspace{4cm}} \;.$$

   However if __________________ , the geometric series is divergent.

   (b) The series in Exercise 1, above, is a geometric series. Is it convergent? If so, what is its sum?

3. Consider the following geometric series:

   $$3 \;-\; 4 \;+\; \frac{16}{3} \;-\; \frac{64}{9} \;+\; \ldots$$

   (a) Find the common ratio (denoted r in the text.)

   (b) Write the series in sigma notation.

(c) Does the series converge? If so, what is its sum?

4. Consider the following geometric series:

$$10 - 2 + 0.4 - 0.08 + \dots$$

(a) Find the common ratio.

(b) Write the series in sigma notation.

(c) Does the series converge? If so, what is its sum?

5. Read Example 8.

(a) Write the harmonic series below.

(b) According to Example 8, does the harmonic series converge or diverge?

3. Test for Divergence

A sequence whose terms do not approach zero has no chance of converging. This is formalized in Theorem 6 and the Test for Divergence.

Exercises

6. Find the Test for Divergence (Theorem 7) and write it below.

7. Consider the series

$$2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \dots$$

   (a) Write a formula for the n^{th} term, a_n, of this series.

   $a_n \quad =$

   (b) Write the series in sigma notation.

   (c) Find the limit of the terms of the series:

   $\lim_{n \to \infty} a_n \quad =$

   (d) Apply the Test for Divergence to show that the series diverges.