Math 132, Prep 11.2B (Finding the Sum of a Series)

Name: ______________________________ **Section:** __________

Names of collaborators: ______________________________

Main Points:

1. evaluating the limit of partial sums (geometric, telescoping)
2. series that can be expressed in terms of simpler ones

1. Evaluating the Limit of Partial Sums

Recall that the sum of a series is the limit of partial sums (if the limit exists), i.e.

$$S = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{n=1}^{N} a_n$$

Usually, it is very difficult to find a formula for the N^{th} partial sum of a series, but in a few cases it can be done: geometric series (as in Example 2) and telescoping series (as in Example 7).

Exercises.

1. Consider the series

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$$

(a) Find a formula for the n^{th} term of the series. Write it in the form $a_n = a \cdot r^{n-1}$.

$a =$ $\qquad\qquad$ $r =$

$a_n =$

(b) Reread Example 2. Use Equation 3 (near the top of page 706) to write a formula for the N^{th} partial sum.

$S_N =$

(c) Evaluate the limit of partial sums, if the limit exists:

$\lim_{N\to\infty} S_N =$

(d) What is the sum of the series?

2. Consider the series $1 - 1 + 1 - 1 + \ldots$.

   (a) Find a formula for the n^{th} term a_n of the series.

   $$a_n \quad =$$

   (b) Write the series in sigma notation.

   (c) Find the first four partial sums of the series.

   (d) Find a formula for the N^{th} partial sum of the series:

   $$S_N \quad = \quad \begin{cases} \underline{\qquad\qquad} & \text{if } N \text{ is } \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & \text{if } N \text{ is } \underline{\qquad\qquad} \end{cases}$$

   (e) Evaluate the limit of partial sums, if the limit exists:

   $$\lim_{N\to\infty} S_N \quad =$$

   (f) Does the series have a sum? If so, what is it?

3. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$. (See Example 7.)

   (a) Use a partial fraction decomposition to rewrite the terms of the series in the form:

   $$\frac{2}{n^2+4n+3} = \frac{A}{n+1} + \frac{B}{n+3}$$

   i.e. find suitable A and B.

   (b) Write out, but do not calculate $S_1, \ldots S_4$. (See Example 7.)

   $S_1 =$

   $S_2 =$

   $S_3 =$

   $S_4 =$

   (c) Now cancel terms to "simplify" but not calculate $S_1, \ldots, S_4$.

   $S_1 =$

   $S_2 =$

   $S_3 =$

   $S_4 =$

   (d) Find a formula for the N^{th} partial sum of the series:

   $S_N =$

(e) Evaluate the limit of partial sums, if the limit exists:

$$\lim_{N\to\infty} S_N \quad =$$

(f) Does the series have a sum? If so, what is it?

2. Series that can be expressed in terms of simpler ones

Theorem 8 describes legitimate manipulations of convergent series. This can be useful for finding the sum of a series that can be rewritten as a sum, difference, or constant multiple of a known convergent series.

Exercises.

4. State Theorem 8. (Make sure to include the words, not just the formulas!)

Another useful fact, related to Theorem 8, pertains to constant multiples of divergent series:

> **Fact.** If $\sum a_n$ is a divergent series and c is a nonzero constant, then $\sum ca_n$ is divergent.

The Note after Example 10 is also useful for determining the convergence or divergence of a series that is similar to one whose convergence (or divergence) is known.

5. Explain the Note after Example 10 in your own words.

6. Determine whether the series is convergent or divergent. If convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3^{n-1}} + \frac{3}{2^{n-1}}\right)$

(b) $\displaystyle\sum_{n=3}^{\infty}\left(\frac{2}{3^{n-1}} + \frac{3}{2^{n-1}}\right)$

(c) $\dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{9} + \dots$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{3+(-1)^n}{2(3^n)}$