Math 132, Review/Prep 11.7 (Practice with Sequences and Series)

Name: ____________________

Names of collaborators: ____________________

Remainder Estimate for Integral Test: $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$

Alternating Series Estimation Theorem: $|R_n| = |s - s_n| \leq |a_{n+1}|$

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1. **Basic Facts and Concepts**
   (a) Sequences of the form $a_n = r^n$ (which of these converge; when convergent, what it the limit)
   (b) Geometric series: $\sum ar^n$ (which of these converge; when convergent, what is the sum)
   (c) Harmonic series and p-series: $\sum \frac{1}{n^p}$ (which of these converge)
   (d) Limit of *terms* of series vs. limit of *partial sums* of series (and Test for Divergence)
   (e) Statement of tests for convergence and divergence
   (f) Absolute and conditional convergence

2. **Sequences.**
   For each sequence below, write out the first five terms. Then find the limit of the sequence, if it exists.
   (a) $a_n = \dfrac{(2n-1)!}{(2n+1)!}$, $n \geq 1$
   (b) $b_n = n^2 e^{-n}$, $n \geq 0$
   (c) $c_1 = 4, \quad c_{n+1} = c_n/3, \; n \geq 0$

3. **Finding the Sum of a Series.**
   (a) Find the sum of the series: $\displaystyle\sum_{n=2}^{\infty} 16 \left(\frac{-3}{4}\right)^n$, if the sum exists.
   (b) Consider $\displaystyle\sum_{k=2}^{\infty} \ln\left(\frac{k}{k+1}\right)$.
      i. Find the limit of the sequence of terms $\{a_k\}$.
      ii. List the first five terms in the sequence of partial sums $\{s_k\}$.
      iii. Find a closed formula for s_k. (Hint: Use the fact that $\ln(a/b) = \ln(a) - \ln(b)$.)
      iv. Evaluate $\lim_{k\to\infty} s_k$ to find the sum of the series, if it exists.

4. **Convergence Tests**

   Use a convergence test of your choice to determine whether the following series converge or diverge. Make a careful argument to justify your answers.

   (a) $\displaystyle\sum_{n=0}^{\infty} ne^{-n^2}$

   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{2^{3k-1}}{(2k-3)!}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{9n^3+n}}$

5. **Absolute and Conditional Convergence**

   Determine whether the following series converge or diverge. In the case of convergence, state whether the convergence is conditional or absolute. Make sure that all of your conclusions are well-supported with careful arguments.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,2^n}{n!}$

   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k}{1+2^k}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$

6. **Estimating the Sum of a Series**

   (a) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$. The first ten partial sums (rounded to two decimal places) are given in the table below.

n	1	2	3	4	5	6	7	8	9	10
S_n	1.00	1.25	1.36	1.42	1.46	1.49	1.51	1.53	1.54	1.55

   One of your classmates conjectures that the sum of the series (rounded to two decimal places) is 1.66. Use the Remainder Estimate for the Integral Test to explain why this is impossible.

   (b) Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$. The first nine partial sums (rounded to three decimal places) are given in the table below.

n	1	2	3	4	5	6	7	8	9
S_n	1.000	0.750	0.861	0.799	0.839	0.811	0.831	0.816	0.828

   One of your classmates conjectures that the sum of the series (rounded to three decimal places) is 0.817. Use the Alternating Series Estimation Theorem to explain why this is impossible.