Name: ______________________ **Section:** ________

Names of collaborators: ______________________

Main Points:

1. Creating a direction field for a differential equation of the form $\frac{dy}{dx} = f(x, y)$ and sketching solution curves.
2. Using Euler's method to sketch solution curves.

1. Direction Fields

Given a differential equation of the form $\frac{dy}{dx} = f(x, y)$, we may create a direction field in the following way: choose points (x, y) in the xy-plane that form a grid; for each point $(x, y) = (a, b)$ in your list, calculate $\frac{dy}{dx}$ by finding $f(a, b)$; finally, at each point $(x, y) = (a, b)$, draw a small line segment whose slope is $m = f(a, b)$. The result is called a **direction field** or a **slope field**.

Exercises.

1. Consider the differential equation $\frac{dy}{dx} = x - y + 1$.

   (a) Complete the table below by choosing x and y-values and finding $\frac{dy}{dx}$ at those values using the differential equation. For example, with $x = 0$ and $y = 0$, we have $\frac{dy}{dx} = 0 - 0 + 1 = 1$.

   (b) Sketch a slope field for this differential equation, by drawing a line segment with the appropriate slope at each point. (E.g. at $(0, 0)$, draw a line segment with slope 1.)

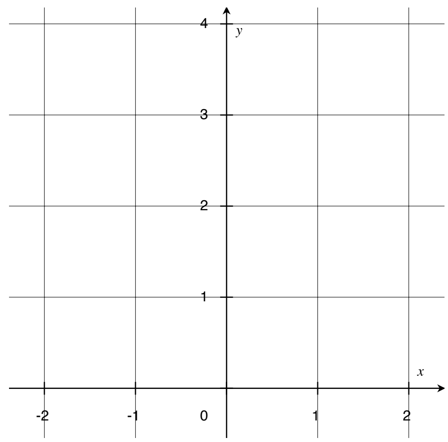
   (c) Use your slope field to sketch three significantly different solution curves.

x	y	$\frac{dy}{dx}$
0	0	1
0	1	0
0	2	-1
0	3	
0	4	
0	-1	
0	-2	
0	-3	
-1	0	
-1	1	

2. Use technology to obtain a direction field for the differential equation $y' = \tan(\frac{1}{2}\pi y)$.

   (a) On the axes below, sketch the graphs of the solutions that satisfy the given initial conditions.

   (i) $y(0) = 1$ (ii) $y(0) = 0.2$ (iii) $y(0) = 2$ (iv) $y(1) = 3$



   (b) Find all the equilibrium solutions. (An equilibrium solution is a solution of the form $y = c$ for some constant c.)

3. Answer the questions below.

3. Figure 10.19 is the slope field for the equation $y' = x+y$.

(a) Sketch the solutions that pass through the points

(i) $(0,0)$ (ii) $(-3,1)$ (iii) $(-1,0)$

(b) From your sketch, guess the equation of the solution passing through $(-1,0)$.

(c) Check your solution to part (b) by substituting it into the differential equation.

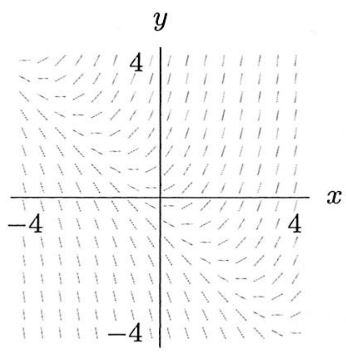


Figure 10.19: Slope field for $y' = x + y$

4. Answer the question below, and **give reasons** for your choices.

6. Match the slope fields in Figure 10.21 with their differential equations:

(a) $y' = -y$ **(b)** $y' = y$ **(c)** $y' = x$
(d) $y' = 1/y$ **(e)** $y' = y^2$

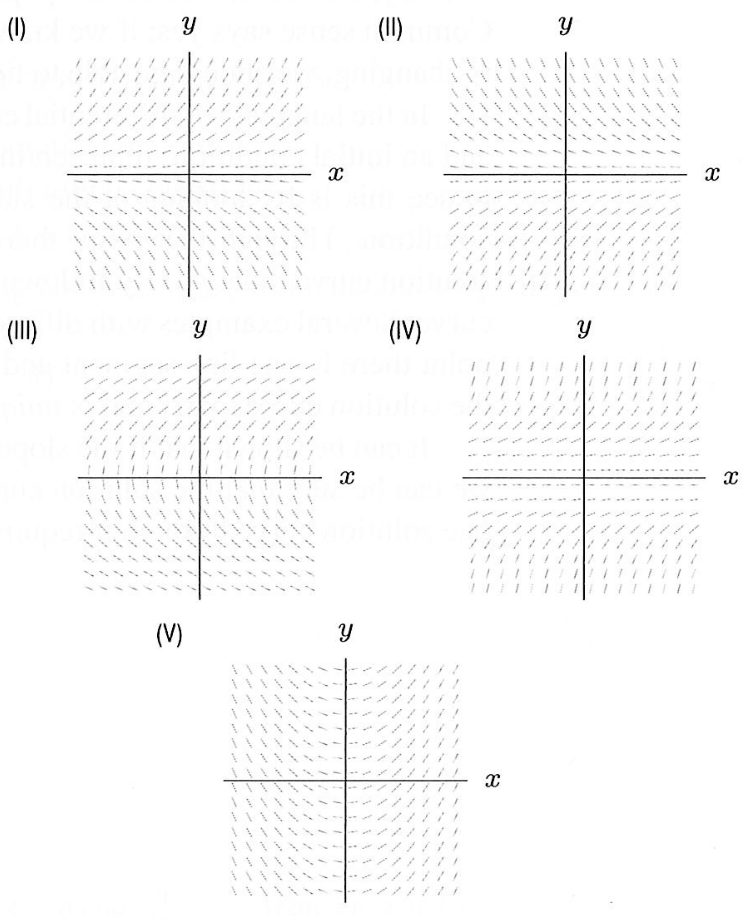


Figure 10.21

2. Euler's Method

Euler's method is an iterative numerical procedure for estimating the values of a particular solution to a differential equation of the form $\frac{dy}{dx} = f(x,y)$.

Given an initial value problem, $\frac{dy}{dx} = f(x,y)$; $y(x_0) = y_0$, we may estimate values of the solution in the following way. First choose a **step size**, h, which is a small change in x. To estimate $y(x_0+h)$, use a linear approximation:

$$y(x_0+h) \quad \approx \quad y_0 + (\tfrac{dy}{dx}|_{(x_0,y_0)})h \ .$$

Call this approximate value y_1. We continue in this way: for $n \geq 1$, let $y_{n+1} = y_n + (\frac{dy}{dx}|_{(x_n,y_n)})h$.

Exercises.

5. Consider the initial value problem:

$$\frac{dy}{dx} \quad = \quad x - y + 1 \ ; \quad y(-3) = 2 \ .$$

We will use Euler's method with step size $h = .5$ to approximate values of the solution.

(a) Our initial value is $y_0 = 2$; this corresponds to $x_0 = -3$. Since our step size is $h = 0.5$, our $x_1 = -3 + 0.5 = -2.5$. Approximate $y(-2.5)$ using the linear approximation:

$$y(-2.5) \quad \approx \quad y(-3) \ + \ \big(\tfrac{dy}{dx}|_{(-3,2)}\big)(0.5)$$

The value you compute will be y_1.

(b) Our x_2 is found simply by adding the step size to x_1: $x_2 = -2.5 + 0.5 = -2$. Using the value you computed for y_1, compute y_2,

$$y_2 \quad = \quad y_1 \ + \ \big(\tfrac{dy}{dx}|_{(-2.5,y_1)}\big)(0.5) \ .$$

(c) Continue this process to compute (x_n, y_n) for $n = 3, 4, 5, 6, 7, 8, 9, 10$. (Do at least one more by hand, and then you may use technology to speed up the process.)

(d) Using the values you computed, fill out the table below, and plot the points to obtain a sketch of the solution curve.

n	x_n	y_n
0	-3	2
1	-2.5	
2	-2	
3		
4		
5		
6		
7		
8		
9		
10		