Math 132, Basic Prerequisite Material

Name: ______________________ **Collaborators:** ______________________

Trigonometry

1. Fill in the following table, using the five standard angles in the first quadrant.

Angle, θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
deg	rad			

2. Sketch the graphs of sine, cosine, and tangent from -2π to 2π. Make sure you have the x-intercepts, max and min values (for sine and cosine), and vertical asymptotes (for tangent).

3. Write each trig function in terms of $\sin x$, $\cos x$, and/or $\tan x$.

$\sec x$ = __________ $\csc x$ = __________

$\tan x$ = __________ $\cot x$ = __________

4. Make sure to know how to evaluate the trig functions at angles in all four quadrants. For example:

$\sin\frac{3\pi}{4}$ = __________ $\sec\frac{7\pi}{6}$ = __________ $\tan\frac{5\pi}{3}$ = __________

$\csc(\frac{-\pi}{4})$ = __________ $\cot\frac{11\pi}{6}$ = __________ $\cos(-\frac{\pi}{2})$ = __________

5. State the Pythagorean Identity (for sine and cosine).

Exponential Growth and Decay

6. Make sure to know the domain, range, y-intercept, horizontal asymptote, and end behavior of the function $f(x) = e^x$.

   Sketch and label graphs of e^x and e^{-x}. Make sure to include the y-intercepts, horizontal asymptotes, and end behavior.

The Natural Logarithm

7. Evaluating the natural logarithm:

   $\ln(1) =$ ________ $\quad \ln(e) =$ ________ $\quad \ln(e^2) =$ ________ $\quad \ln(\sqrt{e}) =$ ________

8. Make sure to know the domain, range, x-intercept, vertical asymptote, and end behavior of the function $f(x) = \ln x$.

   Sketch and label the graphs of $\ln(x)$ and $\ln|x|$. Make sure to include x-intercepts, vertical asymptotes, and end behavior.

   Recall: For a function $f(x)$, the graph of $y = f(|x|)$ is made up of two pieces: the graph of $y = f(x)$ for $x \geq 0$ and its mirror image across the y-axis.

Derivatives of Simple Functions.

- **Constant Functions**: c (not depending on x). Examples $f(x) = 5$, $g(x) = e$, $h(x) = \ln(2)$.

$\frac{d}{dx} c \quad =$

- **Power Functions**: x^a. Examples: x^3, $x^{2/3} = \sqrt[3]{x^2}$, $x^{-4} = 1/x^4$.

$\frac{d}{dx} x^a \quad =$

- **Exponential and Logarithmic Functions**: b^x and $\log_b x$, for $b > 0$; especially when $b = e$.

$\frac{d}{dx} e^x \quad =$

$\frac{d}{dx} b^x \quad =$

$\frac{d}{dx} \ln x \quad =$

$\frac{d}{dx} \log_b x \quad =$

- **Trigonometric Functions**:

$\frac{d}{dx} \sin x \quad =$

$\frac{d}{dx} \cos x \quad =$

$\frac{d}{dx} \tan x \quad =$

$\frac{d}{dx} \sec x \quad =$

$\frac{d}{dx} \csc x \quad =$

$\frac{d}{dx} \cot x \quad =$

- **Inverse Trigonometric Functions**:

$\frac{d}{dx} \sin^{-1} x \quad = \quad \frac{d}{dx} \arcsin x \quad =$

$\frac{d}{dx} \tan^{-1} x \quad = \quad \frac{d}{dx} \arctan x \quad =$

Differentiation Rules. Suppose c is a constant and f and g are differentiable functions.

- **Constant Multiple Rule, Sum and Difference Rules**:

$\frac{d}{dx}\, c \cdot f(x) \quad =$

$\frac{d}{dx} \left(f(x) \pm g(x)\right) \quad =$

- **Product and Quotient Rules**:

$\frac{d}{dx}\, f(x) \cdot g(x) \quad =$

$$\frac{d}{dx} \frac{f(x)}{g(x)} \quad =$$

- **Chain Rule**:

$\frac{d}{dx}\, f(g(x)) \quad =$

Simple Antiderivatives.

- **Constant Functions**: c (not depending on x),

$\int c\, dx \quad =$

- **Some Power Functions**: x^a $(a \neq -1)$,

$\int x^a\, dx \quad =$

- **Exponential Functions**: b^x, for $b > 0$,

$\int e^x \, dx \quad =$

$\int b^x \, dx \quad =$

- **The Reciprocal Function**: $1/x = x^{-1}$

$\int \frac{1}{x} \, dx \quad =$

- **Trigonometric Functions**:

$\int \sin x \, dx \quad =$

$\int \cos x \, dx \quad =$

$\int \sec^2 x \, dx \quad =$

$\int \csc^2 x \, dx \quad =$

$\int \sec x \tan x \, dx \quad =$

$\int \csc x \cot x \, dx \quad =$

- **Derivatives of Inverse Trigonometric Functions**:

$$\int \frac{1}{\sqrt{1-x^2}} \, dx \quad =$$

$$\int \frac{1}{1+x^2} \, dx \quad =$$

Simple Substitution (Undoing the Chain Rule): $\int f(w(x))\, w'(x)\, dx \ = \ \int f(w)\, dw$

- $\int (3x+4)^8 \ dx \ =$

- $\displaystyle\int \frac{1}{1-2x} \ dx \ =$

- $\displaystyle\int \frac{x}{1+x} \ dx \ =$

- $\int_0^{\sqrt{\pi}} x \sin(x^2) \ dx \ =$

- $\displaystyle\int \frac{2x}{(x^2+1)^2} \ dx \ =$

- $\int_0^{\pi/2} \sin^4(x) \cos(x) \ dx \ =$