Math 200-01/05, Multivariable Calculus, Fall 2016
MWF 8:15-9:20 OSS 227 (Section 01), 12:15-1:20 OSS 313 (Section 05)

Instructor: Amy DeCelles

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Course Prerequisites: Successful completion (C- or better) of Math 114, or its equivalent.

Credits and Workload Expectations: 4 credits: 8-10 hours per week outside the classroom.

Course Materials and Resourses:

- Textbook: *Calculus: Single and Multivariable*, 6th ed., Hughes-Hallett, McCallum, et. al.
- *Mathematica* Software: https://www.stthomas.edu/irt/students/desktopsupport/software/
- Math Resource Center (MaRC, OSS 235) : free drop-in peer tutoring, group study areas, solution manuals, WebAssign tutorials, *Mathematica* help, ...

Course Objectives:

- Gaining factual knowledge, terminology and methods (representations of multivariable functions, vectors and vector operations, partial differentiation, multivariate integration, vector fields, line integrals, flux integrals, etc.)
- Learning fundamental principles, generalizations, and theories (linearity and local linearity in several variables, Green's Theorem, Divergence Theorem, Stokes' Theorem, etc.)
- Learning to apply course material (optimization of multivariate functions, volumes of solids, physical applications of multivariate and vector calculus)
- Developing skill in expressing myself orally or in writing (clear written solutions and oral presentations of problems)

Assignments: To prepare for class you will be assigned reading, along with **reading questions** and **discussion problems** related to the reading. You will also be assigned **practice problems** related to material that we have already discussed in class. For each topic, you will also write up one **quality solution**. Occasionally, in-class participation or presentations will be graded.

Late Work: Late work is typically not accepted. The lowest three scores in each assignment category will be dropped at the end of the semester. Extensions may be granted if requested before the due date, and work may certainly be submitted before the due date, if arrangements have been made with the professor in advance. If there is a serious, unforeseeable reason for missing more than three days of class, it is the student's responsibility to contact the professor as

soon as possible and to make appointments with the professor and with Academic Counseling upon returning to classes to make a plan for making up missed work.

Missed Exams: Make-up midterm exams may be given to students with legitimate excuses such as serious illness, university sponsored events, etc., as long as the make-up exam can be taken within a reasonable time frame. If it is not possible to schedule a make-up exam within a reasonable time frame, the grade for the midterm may be prorated from the final exam. Written documentation may be required. Rescheduling the final is not possible except under very extreme circumstances.

Incompletes: Grades of I are normally not given in this course. However, they may be granted due to extenuating circumstances especially if (i) the majority of the course work has been completed at a level of C or better and (ii) the student demonstrates the ability to complete the remaining coursework outside of the classroom. In such cases, a well-documented petition should be submitted to the professor before the last day of classes. Please see the university policies on withdrawals and incomplete grades.

Final Course Grade: The overall score for this course will be computed as outlined below. Final letter grades will be assigned based on the overall score, with the two major components, quality solutions and exams also being considered separately. In particular, the final letter grade will not be higher than one letter grade above the level of the work on written solutions or the work on exams. Exceptional performance on the final may also be taken into account.

- Assignments (45%): reading questions (5%), discussion preparation (5%), discussion/practice problems (5%), and quality solutions and oral presentations (30%)
- Quizzes (10%): six ten-minute quizzes throughout the semester
- Midterm Exams (20%): tentatively Wed Oct 12 and Fri Nov 18
- Final Exam (20%): cumulative; **1:30-3:30 pm Tues Dec 20**, location TBA
- Best Exam (5%): at the end of the semester the score for the best exam will contribute an extra 5% towards the overall score

Disability Accommodations: Academic accommodations will be provided for qualified students with documented disabilities including but not limited to mental health diagnoses, learning disabilities, Attention Deficit Disorder, chronic medical conditions, visual, mobility, and hearing disabilities. Students are invited to contact the Disability Resources office about accommodations early in the semester. Appointments can be made by calling 651-962-6315 or in person in Murray Herrick, room 110. For further information, you can locate the Disability Resources office on the web at http://www.stthomas.edu/enhancementprog/.

Math 200, F2016, Detailed Schedule

Mon	Wed	Fri
Sep 5, 2016	Sep 7, 2016	Sep 9, 2016
Labor Day	Intro to course Due today: RQ 12.1 Next class: RQ 12.2-3, D 12.1	Functions of two variables (12.1) Due today: RQ 12.2-3, D 12.1 Next class: RQ 12.4, D 12.2-3, P&QS 12.1
Sep 12, 2016	Sep 14, 2016	Sep 16, 2016
Graphs, surfaces, contour diagrams (12.2, 12.3) Due today: RQ 12.4, D 12.2-3, P&QS 12.1 Next class: RQ 12.5-6, D 12.4, P&QS 12.2-3	Linear functions (12.4) **Quiz 1** Due today: RQ 12.5-6, D 12.4, P&QS 12.2-3 Next class: RQ 13.1-2, D 12.5-6, P&QS 12.4	Functions of 3 vars (12.5) Limits and continuity (12.6) Due today: RQ 13.1-2, D 12.5-6, P&QS 12.4 Next class: RQ 13.3, D 13.1-2, P&QS 12.5-6
Sep 19, 2016	Sep 21, 2016	Sep 23, 2016
Displacement vectors, vectors in general (13.1, 13.2) Due today: RQ 13.3, D 13.1-2, P&QS 12.5-6 Next class: RQ 13.4, D 13.3, P&QS 13.1-2	The dot product (13.3) Due today: RQ 13.4, D 13.3, P&QS 13.1-2 Next class: RQ 14.1-2, D 13.4, P&QS 13.3	The cross product (13.4) Due today: RQ 14.1-2, D 13.4, P&QS 13.3 Next class: RQ 14.3, D 14.1-2, P&QS 13.4
Sep 26, 2016	Sep 28, 2016	Sep 30, 2016
The partial derivative (14.1, 14.2) Due today: RQ 14.3, D 14.1-2, P&QS 13.4 Next class: RQ 14.4, D 14.3, P&QS 14.1-2	Local linearity,the differential (14.3) **Quiz 2** Due today: RQ 14.4, D 14.3, P&QS 14.1-2 Next class: RQ 14.5, D 14.4, P&PS 14.3	Gradients, the directional derivative in the plane (14.4) Due today: RQ 14.5, D 14.4, P&QS 14.3 Next class: RQ 14.6, D 14.5, P&QS 14.4
Oct 3, 2016	Oct 5, 2016	Oct 7, 2016
Gradients, the directional derivative in 3-space (14.5) Due today: RQ 14.6, D 14.5, P&QS 14.4 Next class: RQ 14.7, D 14.6, P&QS 14.5	The chain rule (14.6) Due today: RQ 14.7, D 14.6, P&QS 14.5 Next class: RQ 14.8, D 14.7, P&QS 14.6	Second-order partial derivs (14.7) Due today: RQ 14.8, D 14.7, P&QS 14.6 Next class: D Rev, P&QS 14.7
Oct 10, 2016	Oct 12, 2016	Oct 14, 2016
Review (12.1-14.7) Due today: D Rev, P&QS 14.7 Next class: Study for exam	**Exam 1** Next class: RQ 15.1, D 14.8	Differentiability (14.8) Due today: RQ 15.1, D 14.8 Next class: RQ 15.2, D 15.1, P 14.8
Oct 17, 2016	Oct 19, 2016	Oct 21, 2016
Critical points: local extrema and saddle points (15.1) Due today: RQ 15.2, D 15.1, P 14.8 Next class: RQ 15.3, D 15.2, P&QS 15.1	Optimization (15.2) Due today: RQ 15.3, D 15.2, P&QS 15.1 Next class: RQ 8.3A, D 15.3, P&QS 15.2	Constrained optimization: Lagrange multipliers (15.3) Due today: RQ 8.3A, D 15.3, P&QS 15.2 Next class: RQ 8.3B, D 8.3A, P&QS 15.3
Oct 24, 2016	Oct 26, 2016	Oct 28, 2016
Polar coordinates and graphs (8.3A) **Quiz 3** Due today: RQ 8.3B, D 8.3A, P&QS 15.3 Next class: RQ 16.1-2, D 8.3B, P&QS 8.3A	Calculus with polar coords. (8.3B) Due today: RQ 16.1-2, D 8.3B, P&QS 8.3A Next class: RQ 16.3, D 16.1-2, P&QS 8.3B	Fall Break
Oct 31, 2016	Nov 2, 2016	Nov 4, 2016
Double integrals (16.1, 16.2) Due today: RQ 16.3, D 16.1-2, P&QS 8.3B Next class: RQ 16.4, D 16.3, P&QS 16.1-2	Triple Integrals (16.3) Due today: RQ 16.4, D 16.3, P&QS 16.1-2 Next class: RQ 16.5, D 16.4, P&QS 16.3	Double integrals in polar coordinates (16.4) Due today: RQ 16.5, D 16.4, P&QS 16.3 Next class: RQ 21.2, D 16.5, P&QS 16.4
Nov 7, 2016	Nov 9, 2016	Nov 11, 2016
Integrals in cylindrical and spherical coordinates (16.5)) **Quiz 4** Due today: RQ 21.2, D 16.5, P&QS 16.4 Next class: RQ 17.1-2, D 21.2, P&QS 16.5	Change of variables and Jacobian (21.2) Due today: RQ 17.1-2, D 21.2, P&QS 16.5 Next class: RQ 17.3, D 17.1-2, P&QS 21.2	Parametrized curves and motion in 3-space (17.1, 17.2) Due today: RQ 17.3, D 17.1-2, P&QS 21.2 Next class: RQ 18.1-2, D 17.3, P&QS 17.1-2
Nov 14, 2016	Nov 16, 2016	Nov 18, 2016
Vector fields (17.3) Due today: RQ 18.1-2, D 17.3, P&QS 17.1-2 Next class: D Rev, P&QS 17.3	Review (14.8-17.3) Due today: D Rev, P&QS 17.3 Next class: Study for exam	**Exam 2** Next class: RQ 18.3, D 18.1-2
Nov 21, 2016	Nov 23, 2016	Nov 25, 2016
Line integrals (18.1, 18.2) Due today: RQ 18.3, D 18.1-2 Next class: RQ 18.4, D 18.3, P&QS 18.1-2	Mathematica Project	Thanksgiving Break
Nov 28, 2016	Nov 30, 2016	Dec 2, 2016
Gradient fields, path-independent fields (18.3) Due today: RQ 18.4, D 18.3, P&QS 18.1-2 Next class: RQ 19.1-2, D 18.4, P&QS 18.3	Path-dependent fields, Green's theorem (18.4) Due today: RQ 19.1-2, D 18.4, P&QS 18.3 Next class: RQ 19.3, D 19.1-2, P&QS 18.4	Flux integrals (19.1, 19.2) **Quiz 5** Due today: RQ 19.3, D 19.1-2, P&QS 18.4 Next class: RQ 19.4, D 19.3, P&QS 19.1-2
Dec 5, 2016	Dec 7, 2016	Dec 9, 2016
Divergence of a vector field (19.3) Due today: RQ 19.4, D 19.3, P&QS 19.1-2 Next class: RQ 20.1, D 19.4, P&QS 19.3	Divergence theorem (19.4) Due today: RQ 20.1, D 19.4, P&QS 19.3 Next class: RQ 20.2, D 20.1, P&QS 19.4	Curl of a vector field (20.1) Due today: RQ 20.2, D 20.1, P&QS 19.4 Next class: RQ 20.3, D 20.2, P&QS 20.1
Dec 12, 2016	Dec 14, 2016	Dec 16, 2016
Stokes' theorem (20.2) **Quiz 6** Due today: RQ 20.3, D 20.2, P&QS 20.1 Next class: D 20.3, P&QS 20.2	Three fundamental theorems (20.3) Due today: D 20.3, P&QS 20.2 Due 11:59 tonight: P&QS 20.3	

Math 200, F 2016, Assignments from Textbook

Section	Discussion	Practice	QS
12.1	2, 4, 11, 12, 21, 22, 23, 25	13, 14, 27, 28, 31	24
12.2	4, 5, 6, 9, 10, 16, 17, 18	2, 3, 7, 8, 11, 31	18
12.3	5, 7, 16, 25	6, 8, 9, 17, 28	
12.4	3, 6, 7, 8, 9, 10, 11, 15, 25, 26	4, 5, 12, 14, 18, 19, 20, 27, 28	16
12.5	1, 8, 11, 12, 13, 20, 37	2, 9, 10, 21, 35	36
12.6	1, 2, 5, 19	3, 4, 6, 18	
13.1	3, 5, 7, 8, 10, 26, 34, 37	4, 6, 20-24, 33	32
13.2	8, 9, 11, 16	12, 13, 24, 26	
13.3	10, 15, 19, 29, 32, 42, 49, 53	11, 13, 27, 30, 40, 43, 48, 59	36
13.4	20, 21, 22, 29, 31, 32	25, 26, 27, 30, 35	34
14.1	4, 9, 10, 11, 12, 13, 21, 25	6, 8, 24	
14.2	5, 10, 12, 13, 23, 45	8, 15, 18, 24, 25, 49	44
14.3	3, 6, 11, 17, 22, 30	2, 5, 16, 25, 35	18
14.4	4, 9, 18, 23, 34, 35, 38, 51, 82	7, 10, 25, 36, 37, 39, 40, 84	50
14.5	10, 12, 17, 20, 21, 24, 25, 54	3, 5, 8, 18, 22, 26, 39	48
14.6	4, 5, 9, 12, 17, 18	2, 6, 10, 13, 20	14
14.7	4, 5, 15, 21, 22, 37, 39, 41	6, 7, 14, 20, 27, 40, 44, 51	18
14.8	1, 11	12	
15.1	9, 12, 13, 33a	11, 16, 32a	10
15.2	5, 7, 9, 10, 14, 19	6, 8, 11, 17	20
15.3	5, 13, 18, 19	3, 8, 14, 32	12
8.3A	9, 10, 11, 14, 17	12, 13, 15, 18, 19	16
8.3B	21, 23, 26, 27, 35	22, 24, 28, 30, 31, 33	32
16.1	3	4	
16.2	13, 14, 28, 29, 33, 35, 38	9, 30, 34, 39, 41	42
16.3	28, 31, 45, 65	43, 48, 66	50
16.4	16, 19, 27, 30	17, 20, 29	28
16.5	3, 6, 40, 49, 54, 57	2, 5, 48, 58	52
21.2	3, 10, 13, 18	11, 14, 15	16
17.1	57, 61, 67	56, 58, 71	
17.2	22, 41	26, 43	28
17.3	7, 10, 20, 26, 37	8, 9, 27, 29	28
18.1	37	38, 42	
18.2	7, 18, 22, 28, 36	8, 14, 29	12
18.3	7, 8, 14, 17, 54	15, 19, 24, 48	30
18.4	1, 5, 12, 22, 31	13, 23	20
19.1	59, 61ac	62ac	
19.2	9, 17, 25, 36	33, 41, 46, 50	38
19.3	4, 10, 16, 17, 27	8, 18, 23, 32, 33	28
19.4	1, 5, 6, 15, 16	3, 12, 17, 20, 31	18
20.1	6, 7, 8, 16, 19, 21	18, 23, 28	22
20.2	2, 3, 12, 28	13, 15, 16, 27	14
20.3	4, 13, 14, 17, 21	5, 10, 18, 24	12

Main Points:

1. Functions of two variables: formulas, tables, contour diagrams
2. Three-space, graphs of two-variable functions in three-space

Overview

A function is a mathematical object that describes the dependency of one quantity upon one or more other quantities. In particular, a **function of two variables** is one that describes the dependency of one quantity upon *two* other quantities: the elevation of the terrain as it depends on latitude and longitude is an example of a function of two variables. Such a function may be described in several ways, e.g. by a formula, table, or graphical representation.

Recall that Cartesian coordinates x and y are often used to parametrize two-dimensional space; in Section 12.1, we see how the Cartesian coordinates x, y, and z can be used to parametrize three-dimensional space, also called **3-space**, for short.

1. Reading Assignment (for Wednesday September 7)

Read Section 12.1. Words and phrases in *italics* are important words and phrases. Formulas in blue boxes are important formulas. Pay attention to these things and take notes on them in your notebook!

2. Discussion Problems (for Friday September 9)

To prepare for class on Friday, try the following problems from the textbook:

12.1: # 2, 4, 11, 12, 21, 22, 23, 25

You will have a few minutes in class on Friday to discuss the problems with others, while I circulate to check that you have prepared the problems and to answer quick clarification questions. After this, I will ask students (or groups of students) to write solutions on the board. These solutions will form the basis for a class discussion of the problems. The goal is that everyone leaves class with a good understanding of the discussion problems.

3. Practice Problems and Quality Solution (for Monday September 12)

The following problems are for additional practice:

12.1: # 13, 14, 27, 28, 31

Write these problems up neatly, making sure your steps and your final answers are clear. Some of these problems will be graded.

Also for Monday, write up a clearly explained solution to **12.1 # 24**. Imagine that you have a friend who does not understand functions of two variables. To help your friend, you decide to explain functions of two variables by explaining how to solve this particular problem. You should begin by stating the original problem clearly; then use full sentences to explain your solution. Your explanation of this problem will be carefully graded for correctness, thoroughness, readability, and insight.

I will give a more detailed hand-out on turning in written homework in class on Friday.

Main Points:

1. Graphs of two-variable functions in three-space
2. Contour diagrams

Overview

Recall that a **function of two variables** is one that describes the dependency of one quantity upon *two* other quantities. We consider two kinds of graphical representations of functions of two variables: surfaces in three-dimensional space (Section 12.2) and contour diagrams, which look like weather maps or elevation maps (Section 12.3).

Just as a single-variable function $f(x)$ can be represented by its graph $y = f(x)$, which is a curve in two-dimensional space, a two-variable function $f(x, y)$ can be represented by its graph $z = f(x, y)$, which is a surface in three-dimensional space.

1. Reading Assignment (for Friday September 9)

Read Sections 12.2-12.3. Words and phrases in *italics* are important words and phrases. Formulas in blue boxes are important formulas. Pay attention to these things and take notes on them in your notebook! Then answer the reading questions. These are due at the beginning of Friday's class.

2. Discussion Problems (for Monday September 12)

To prepare for class on Monday, try the following problems from the textbook:

12.2: # 4, 5, 6, 9, 10, 16, 17, 18; 12.3: # 5, 7, 16, 25

You will have a few minutes in class on Monday to discuss the problems with others, while I circulate to check that you have prepared the problems and to answer quick clarification questions. After this, I will ask students (or groups of students) to write solutions on the board. These solutions will form the basis for a class discussion of the problems. The goal is that everyone leaves class with a good understanding of the discussion problems.

3. Practice Problems and Quality Solution (for Wednesday September 14)

The following problems are for additional practice:

12.2: # 2, 3, 7, 8, 11, 31; 12.3: # 6, 8, 9, 17, 28

Write these problems up neatly, making sure your steps and your final answers are clear. Some of these problems will be graded.

Also for Wednesday, write up a clearly explained solution to **12.2 # 18**. Imagine that you have a friend who does not understand graphs of functions of two variables. To help your friend, you decide to explain graphs of functions of two variables by explaining how to solve this particular problem. You should begin by stating the original problem clearly; then use full sentences to explain your solution. Your explanation of this problem will be carefully graded for correctness, thoroughness, readability, and insight.

Main Points:

1. Linear functions of two variables
2. The graph of a linear function is a plane in 3-space
3. Tables and contour diagrams representing linear functions of two variables

Overview

A function of two variables is **linear** if the output depends linearly on each input: i.e. a given increase in one input (without changing the other input) results in a constant increase in the output. The graph of a linear function of two variables is a **plane** in three-dimensional space. The equation of a plane in 3-space passing through the point (x_0, y_0, z_0) is of the form:

$$z = z_0 + m(x - x_0) + n(y - y_0)$$

where m and n are constants, representing the slope of the plane in the x-direction and the slope of the plane in the y-direction, respectively.

1. Reading Assignment (for Monday September 12)

Read Section 12.4. Words and phrases in *italics* are important words and phrases. Formulas in blue boxes are important formulas. Pay attention to these things and take notes on them in your notebook! Then answer the reading questions. These are due at the beginning of Monday's class.

2. Discussion Problems (for Wednesday September 14)

To prepare for class on Wednesday, try the following problems from the textbook:

12.4: # 3, 6, 7, 8, 9, 10, 11, 15, 25, 26

You will have a few minutes in class on Wednesday to discuss the problems with others, while I circulate to check that you have prepared the problems and to answer quick clarification questions. After this, I will ask students (or groups of students) to write solutions on the board. These solutions will form the basis for a class discussion of the problems. The goal is that everyone leaves class with a good understanding of the discussion problems.

3. Practice Problems and Quality Solution (for Friday September 16)

The following problems are for additional practice:

12.4: # 4, 5, 12, 14, 18, 19, 20, 27, 28

Write these problems up neatly, making sure your steps and your final answers are clear. Some of these problems will be graded.

Also for Friday, write up a clearly explained solution to **12.4 # 16**. Imagine that you have a friend who does not understand linear functions of two variables. To help your friend, you decide to explain linear functions of two variables by explaining how to solve this particular problem. You should begin by stating the original problem clearly; then use full sentences to explain your solution. Your explanation of this problem will be carefully graded for correctness, thoroughness, readability, and insight.

Main Points:

1. Representing functions of three variables using level surfaces
2. Limits and continuity of functions of two variables

Overview

A **function of three variables** is used to desribe the dependency of a particular quantity on *three* other quantities. Just as the graph of a single-variable function is a curve in two-dimensional space and the graph of a two-variable function is a surface in three-dimensional space, the graph of a three-variable function is a solid in four-dimensional "space." Unfortunately this is impossible to draw. However, we can use the idea of a contour diagram to help us here. Just as the (three-dimensional) graph of a two-variable function can be "projected down" to the plane and represented by a level curves, the (four-dimensional) graph of a three-variable function can be projected down to 3-space and represented by **level surfaces**.

The notions of **limit** and **continuity** from single-variable calculus are important in multi-variable calculus as well. Recall that a single-variable function $f(x)$ is continuous at a point $x = a$ if the values of $f(x)$ approach a specific, finite number (called the limit) as x approaches a and that limit is equal to $f(a)$. Similarly, a function $f(x, y)$ of two variables is continuous at a point (a, b) if as the values of x approach a and the values of y approach b (from any direction!) the values of $f(x, y)$ approach a specific, finite value (the limit) and that limit is equal to $f(a, b)$.

1. Reading Assignment (for Wednesday September 14)

Read Sections 12.5 and 12.6. Words and phrases in *italics* are important words and phrases. Formulas in blue boxes are important formulas. Pay attention to these things and take notes on them in your notebook! Then answer the reading questions. These are due at the beginning of Wednesday's class.

2. Discussion Problems (for Friday September 16)

To prepare for class on Friday, try the following problems from the textbook:

12.5: # 1, 8, 11, 12, 13, 20, 37; 12.6: # 1, 2, 5, 19

You will have a few minutes in class on Friday to discuss the problems with others, while I circulate to check that you have prepared the problems and to answer quick clarification questions. After this, I will ask students (or groups of students) to write solutions on the board. These solutions will form the basis for a class discussion of the problems. The goal is that everyone leaves class with a good understanding of the discussion problems.

3. Practice Problems and Quality Solution (for Monday September 19)

The following problems are for additional practice:

12.5: # 2, 9, 10, 21, 35; 12.6: # 3, 4, 6, 18

Write these problems up neatly, making sure your steps and your final answers are clear. Some of these problems will be graded.

Also for Monday, write up a clearly explained solution to **12.5 # 36**. Imagine that you have a friend who does not understand linear functions of two variables. To help your friend, you decide to explain linear functions of two variables by explaining how to solve this particular problem. You should begin by stating the original problem clearly; then use full sentences to explain your solution. Your explanation of this problem will be carefully graded for correctness, thoroughness, readability, and insight.