**Name:** ______________________ **Section:** __________

**Names of collaborators:** ______________________

This lab contains instructions for using *Mathematica* to answer questions about scalar and vector-valued functions of two variables. The commands you need are introduced one by one. Most of the parts of the problems below simply tell you to enter a command in *Mathematica*. However, some parts of some problems also ask you to answer a question, so make sure to answer all the questions! When you have completed the lab, print off your *Mathematica* work, and staple it to this packet.

**General *Mathematica* tips:**

1. **Capital Letters** Every built-in function (like `Plot`, `Sqrt`, `Sin`) and every built-in constant (like `Pi`) begins with a capital letter.
2. **Parentheses** Use parentheses to specify the order of operations, as when using a graphing calculator.
3. **Square Brackets** Whenever passing an argument to a function (i.e. plugging something in to a function), you must use square brackets. For example, use `Sin[Pi/4]` to find $\sin(\pi/4)$.
4. **Curly Braces** Whenever creating lists or ranges, use curly braces. For example, when using the `Plot` command, use `{x, -10, 10}` to set the $x$-range to $[-10, 10]$.
5. **Shift + Enter** To execute a command, it is necessary to press `Shift + Enter`.

## I. Graphs and Contour Plots of Scalar-Valued Functions of Two Variables.

1. Recall that the behavior of a vibrating guitar string of length $\pi$ can be modeled using the function $f(x,t) = \cos(t)\sin(x)$ for $0 \le x \le \pi$ and $0 \le t \le 2\pi$. Here $f(t,x)$ represents the vertical displacement of a point on the string $x$ units from one end, $t$ units of time after the string has been plucked.

   (a) Use `Plot3D` to obtain a graph of $z = f(x,t)$ in 3-space.

   ```
   Plot3D[Cos[t]Sin[x], {x, 0, Pi}, {t, 0, 2*Pi}]
   ```

   (b) Use the `ContourPlot` command to create a contour diagram for $f(x,t)$.

   ```
   ContourPlot[Cos[t]Sin[x], {x, 0, Pi}, {t, 0, 2*Pi}]
   ```

   Notice that there are no labels on the contour lines.

   (c) To add labels to the contour lines, include the option `ContourLabels -> True` in the `ContourPlot` command:

   ```
   ContourPlot[Cos[t]Sin[x],{x,0,Pi},{t,0,2*Pi},ContourLabels->True]
   ```

   (d) Use the `Plot` command to obtain a graph of $z = f(x,0)$, which represents the one-variable function obtained by fixing $t = 0$ and letting $x$ vary from 0 to $\pi$:

   ```
   Plot[Cos[0]Sin[x], {x, 0, Pi}, PlotRange->{-1,1}]
   ```

   The option `PlotRange->{-1,1}` specifies the $z$-range.

   (e) Use the `Manipulate` command with the `Plot` to obtain graphs of $z = f(x,t)$ as a function of $x$, for a range of $t$-values.

   ```
   Manipulate[Plot[Cos[t]Sin[x],{x,0,Pi}, PlotRange->{-1,1}], {t,0,2*Pi}]
   ```

   Use the slider to allow $t$ to vary. This shows the motion of the guitar string over time.

2. Consider the function $g(x, y) = 2 + 4x + 4x^2 - 4y + y^2$.

   (a) Use `Plot3D` to obtain a graph of $z = g(x, y)$ in 3-space. What kind of surface is this?

   (b) Use `ContourPlot` to obtain a contour diagram for $g(x, y)$ with labeled contours.

## II. Optimizing Scalar-Valued Functions of Two Variables.

3. Consider $f(x, t) = \cos(t)\sin(x)$ on the region $(t - \frac{\pi}{2})^2 + (x - \pi)^2 \leq 2$.

   (a) Use the `RegionFunction` option within the `Plot3D` command to plot $f(x, t)$ on the specified region:

   ```
   RegionFunction -> Function[{t, x, z}, (t - Pi/2)^2 + ((x - Pi)^2) < 2]
   ```

   Note: You need to insert this as an option into the `Plot3D` command. That means you should put it where the " . . . " are in the following:

   ```
   Plot3D[Cos[t] Sin[x], {x, -Pi, Pi}, {t, 0, 2*Pi}, ...]
   ```

   (b) Use the `RegionFunction` option within the `ContourPlot` command to obtain a contour plot for $f(x, t)$ on the specified region. (Make sure the contours are labeled in your contour diagram.)

   ```
   ContourPlot[Cos[t] Sin[x],{x,0,Pi}, {t,0,2*Pi}, ContourLabels->True, ...]
   ```

   (c) Use your contour diagram to estimate the global maximum and minimum values of the function on the specified region.

   Global min value: __________ Location(s) of global min value: ____________________

   Global max value: __________ Location(s) of global max value: ____________________

4. Consider the function $g(x, y) = 2 + 4x + 4x^2 - 4y + y^2$.

   (a) Use the contour diagram from above to estimate the global min value for $g(x, y)$ and its location.

   Global minimum value: __________ Location of global minimum value: __________

   (b) Use the `Minimize` command to find the exact global minimum value and its location.

   ```
   Minimize[2 + 4x + 4x^2 - 4y + y^2, {x, y}]
   ```

   Global minimum value: __________ Location of global minimum value: __________

   (c) Now restrict $g(x, y)$ to the domain $\mathcal{D}$ given by $x^2 + y^2 < 25$. Use `Plot3D` with the `RegionFunction` option to graph $g(x, y)$ on $\mathcal{D}$.

   ```
   RegionFunction -> Function[{x, y, z}, x^2 + y^2 < 25]
   ```

(d) Use `ContourPlot` with the `RegionFunction` option to obtain a contour diagram of $g(x, y)$ on $\mathcal{D}$.

(e) Use the contour diagram to estimate the values and locations of global extrema of $g(x, y)$ on $\mathcal{D}$.

Global min value: __________ Location(s) of global min value: ____________________

Global max value: __________ Location(s) of global max value: ____________________

**II. Vector Fields.**

5. Use `VectorPlot` to depict the vector field $\vec{F}(x, y) = x\hat{\imath} + y\hat{\jmath}$.

```
VectorPlot[{x, y}, {x, -10, 10}, {y, -10, 10}]
```

Use the option `VectorStyle -> Arrowheads[0.02]` to make the arrowheads smaller.

Is the circulation of $\vec{F}$ around the origin positive, negative, or zero? ____________________

6. Use `VectorPlot` to depict the vector field $\vec{F}(x, y) = -y\hat{\imath} + x\hat{\jmath}$.

```
VectorPlot[{-y, x},{x,-10,10},{y,-10,10}, VectorStyle->Arrowheads[0.02]]
```

Is the circulation of $\vec{F}$ around the origin positive, negative, or zero? ____________________

7. (a) Use `VectorPlot` to depict the vector field $\vec{F}(x, y) = y\hat{\imath} + x\hat{\jmath}$.

(b) Use `VectorPlot` to depict the vector field $\vec{F}(x, y) = (y/\sqrt{x^2 + y^2})\hat{\imath} + (x/\sqrt{x^2 + y^2})\hat{\jmath}$. Note that the square root command in *Mathematica* is `Sqrt`.

(c) Compute the gradient of $f(x, t) = \cos(t)\sin(x)$ and use `VectorPlot` to depict it.

8. Print off your *Mathematica* work, and staple it to this packet. Make sure that all of the commands you were asked to enter into *Mathematica* are shown in your print-off.