Math 200, Graphs, Surfaces, Contour Diagrams (Sections 12.2, 12.3)

Name: ______________________________ **Section:** __________

Read Sections 12.2 and 12.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 1 in Section 12.2, and try this similar problem: describe in words the graphs of the function $\ell(x, y) = x^2 + y^2 - 6$.

2. Reread Example 3 in Section 12.2, which describes the graph of $g(x, y) = x^2 - y^2$, and answer these questions about the example.

   (a) What are the cross-sections of the graph of $g(x, y)$ with y fixed?

   (b) What are the cross-sections of the graph of $g(x, y)$ with x fixed?

   (c) What is the shape of the graph of $g(x, y)$?

3. Reread the part in Section 12.2 about linear functions of two variables. What is the shape of the graph of a linear function of two variables?

4. Reread the part in Section 12.2 about cylinders. Describe the graph of $z = y^2$ in 3-space.

5. Reread Examples 3, 5, and 6 in Section 12.3.

   (a) Describe, in words, the contour diagram of a parabolic bowl.

   (b) Describe, in words, the contour diagram of a plane.

   (c) Sketch a contour diagram for a saddle-shaped surface. Include level curves for at least five z-values.

6. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 12.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the part about linear functions from a numerical point of view. Now try Exercises 1 and 3 at the end of the section (page 697). Make sure to explain your reasoning for Exercise 3.

2. Reread the part about contour diagrams of linear functions. Now try Exercises 7 and 8 at the end of the section (page 697). Make sure to explain your reasoning.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Functions of three variables; Limits and continuity (Sections 12.5, 12.6)

Name: ______________________ **Section:** ________

Read Sections 12.5 and 12.6. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Describe, in your own words, what a level surface of a function of three variables is.

2. Consider the function $f(x, y, z) = x^2 + y^2 - z^2$. (For this problem, the catalog of surfaces on page 702 might be useful.)

   (a) What does the level surface corresponding to $f(x, y, z) = 1$ look like? (Hint: the surface will have the equation $x^2 + y^2 - z^2 = 1$.)

   (b) What does the level surface corresponding to $f(x, y, z) = 0$ look like?

   (c) What does the level surface corresponding to $f(x, y, z) = -1$ look like?

3. In your own words, explain why the notions of limit and continuity are more subtle in the multivariable case than the single-variable case. (See the very end of Section 12.6.)

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Displacement vectors and vectors in general (Sections 13.1, 13.2)

Name: ____ **Section:** ________

Read Sections 13.1 and 13.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. (a) Briefly describe how a vector is different from a number by explaining the concepts of magnitude and direction.

   (b) Now try Exercises 1-5 at the end of Section 13.2 (page 732.)

2. Try Exercise 1 at the end of Section 13.1 (page 724).

3. Reread Example 4 in Section 13.1. Now suppose $\vec{w}$ is a vector of length 4, making an angle of $\pi/3$ with the positive x-axis. Resolve $\vec{w}$ into components using sine and cosine.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ________________________ **Section:** ________

Read Section 13.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercises 1, 3, 5, and at the end of Section 13.3 (page 740.)

2. Suppose $\vec{v}$ and $\vec{w}$ are vectors of length 3 and 4 respectively. What can we conclude about $\vec{v}$ and $\vec{w}$ if

   (a) $\vec{v} \cdot \vec{w} = 0$?

   (b) $\vec{v} \cdot \vec{w} = 12$?

   (c) $\vec{v} \cdot \vec{w} = -12$?

3. Reread Examples 8 and 9.

   (a) What percentage of the force of the wind is in the direction of the sailboat's motion?

   (b) What are $\|\vec{F}_{\text{perp}}\|$ and $\|\vec{F}_{\text{parallel}}\|$ in the scenario of Example 9? (You will have to do some computations to find these; they are not stated explicitly in the text.) Which one of these is relevant for computing the work done on the sailboat?

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 13.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercises 1, 3, and 5 at the end of Section 13.4 (page 749.)

2. If $\vec{v}$ and $\vec{w}$ both lie in the xy-plane, what can we say about the direction of $\vec{v} \times \vec{w}$?

3. Reread Example 4. Redo the problem using the displacement vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$. Do you get the same answer?

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, The partial derivative (Sections 14.1, 14.2)

Name: ______________________________ **Section:** __________

Read Sections 14.1 and 14.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Examples 1 and 2 in Section 14.1.

   (a) Explain, in your own words, what $T_x(2,1)$ represents. (Make sure to include units.) The fact that it is positive indicates what about the temperature of the metal plate?

   (b) Explain, in your own words, what $T_y(2,1)$ represents. (Make sure to include units.) The fact that it is negative indicates what about the temperature of the metal plate?

2. Reread Example 4 in Section 14.1.

   (a) Explain, in your own words, what $H_x(10,20)$ represents. (Make sure to include units.) The fact that it is negative indicates what about the temperature in the room?

   (b) Explain, in your own words, what $H_t(10,20)$ represents. (Make sure to include units.) The fact that it is positive indicates what about the temperature in the room?

3. Compute the partial derivatives of $f(x, y) = 2x\sin(y)$ algebraically:

   (a) $f_x(x, y) \quad =$

   (b) $f_y(x, y) \quad =$

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Local linearity, the differential (Section 14.3)

Name: ______________________________ **Section:** __________

Read Section 14.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 1, and try Exercise 1 at the end of the section, page 777.

2. Reread Example 4, and try Exercise 9 at the end of the section, page 777.

3. Reread Example 5.

   (a) What does $f_T(T, P)$ represent? What does dT represent? What does $f_T(T, P)\, dT$ represent? (Make sure to include units.)

   (b) What does $f_P(T, P)$ represent? What does dP represent? What does $f_P(T, P)\, dP$ represent? (Make sure to include units.)

   (c) What does $d\rho$ represent? What are its units?

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Gradients and directional derivatives in the plane (Section 14.4)

Name: ______________________________ **Section:** __________

Read Section 14.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 2, focusing on the third contour diagram, the one for the function $h(x, y)$. Notice that the directional derivatives in the directions of both $\vec{v}$ and $\vec{w}$ at the indicated point are negative.

   (a) Is it possible to find a vector $\vec{u}$ at the point indicated in the diagram such that the directional derivative is positive? If so, what direction would $\vec{u}$ be pointing? If not, why not?

   (b) Is it possible to find a vector $\vec{u}$ at the point indicated in the diagram such that the directional derivative is zero? If so, what direction would $\vec{u}$ be pointing? If not, why not?

2. Reread Example 5, and try Exercise 15 at the end of the section, page 785.

3. Reread Example 7, and use your answer to the previous question to find the directional derivative for $f(x, y) = x^2y + 7xy^3$ at the point $(1, 2)$ in the direction of the vector $\vec{i} - \vec{j}$.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Gradients and directional derivatives in space (Section 14.5)

Name: ______________________ **Section:** ________

Read Section 14.5. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercises 13 and 19 at the end of the section, page 793.

2. Suppose f is a three-variable function, differentiable at the point (a, b, c), and $\text{grad}\, f(a, b, c) \neq 0$. Complete each statement below by filling in the missing words.

   (a) $\text{grad}\, f(a, b, c) \neq 0$ points in the direction of the ... of f.

   (b) $\text{grad}\, f(a, b, c) \neq 0$ is perpendicular to ... of f at (a, b, c)

   (c) The magnitude of $\text{grad} f(a, b, c) \neq 0$ is ... of f at (a, b, c).

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 14.6. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Examples 1, 2, and 3. Try Exercise 1 at the end of the section, page 803.

2. Reread Example 4, and try Exercise 7 at the end of the section, page 803.

3. What application of the general chain rule to science is discussed in this section?

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Second-order partial derivatives (Section 14.7)

Name: ______________________________ **Section:** __________

Read Section 14.7. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 1, and try Exercise 1 at the end of the section, page 812.

2. Reread Example 3 (and look back at Example 4 from Section 14.2, page 767.) Explain, in your own words, what the following quantities represent, in practical terms. Make sure to include units.

   (a) $f_{xx}(x, t)$

   (b) $f_{xt}(x, t)$

   (c) $f_{tx}(x, t)$

   (d) $f_{tt}(x, t)$

3. Reread Example 4.

   (a) What is the function $f(x, y)$ we are approximating in this problem?

   (b) What is the linear approximation of $f(x, y)$ near $(0, 0)$?

   (c) What is the quadratic approximation of $f(x, y)$ near $(0, 0)$?

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Critical points: local extrema and saddle points (Section 15.1)

Name: ______________________________ **Section:** __________

Read Section 15.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. (a) What is a critical point of a multivariate function?

   (b) What are the three kinds of critical points?

2. Try Exercises 2 and 3 at the end of the section, page 836.

3. Consider the function $f(x, y) = x^2 - 2xy + 3y^2 - 8y$.

   (a) Compute grad $f(x, y)$, and find the critical point(s) of $f(x, y)$.

(b) Compute the second-order partial derivatives of $f(x, y)$.

(c) Use the second derivative test for functions of two variables to classify each critical point.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** ________

Read Section 15.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Explain, in your own words, the difference between local and global extrema.

2. In this section we learn some conditions under which a function is guaranteed to have global extrema on a region.

   (a) What is the name of the theorem that gives these conditions?

   (b) What condition does the theorem impose on the function itself?

   (c) What two conditions does the theorem impose on the region?

3. Try Exercises 2 and 3 at the end of the section, page 845.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Constrained optimization: Lagrange multipliers (Section 15.3)

Name: ______________________________ **Section:** __________

Read Section 15.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the introductory example (maximizing production subject to a budget constraint, pages 848-849) carefully.

   (a) In this problem, what is the objective function, namely the quantity we are trying to maximize? (Give the formula for this function, and state what it represents.)

   (b) In this problem, what is the constraint function? (Again, state the formula and its meaning.)

   (c) Reread the last paragraph on page 848. Complete the sentence: The maximum value of the objective function, subject to the constraint, occurs at the point where ...

   (d) Reread page 849, on Lagrange multipliers. At the optimum point, what two vectors are parallel?

2. (a) Reread Example 1, and try Exercise 1, at the end of the section, page 855.

(b) Show that $f(x, y) = x + y$ does not have any local maxima or minima in the interior of the circle, i.e. in the region $x^2 + y^2 < 1$. (Use the methods of 15.1.)

(c) What and where are the global extrema of $f(x, y)$ on the (closed and bounded!) region $x^2 + y^2 \leq 1$?

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read the beginning of Section 8.3, **only up to and including Example 8**. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Examples 1 and 2, then practice converting between Cartesian and polar coordinates by working through Exercises 1-8 at the end of the section, page 438.

2. Reread Example 6. Graph $r = 1 + 2\cos\theta$ yourself by hand by making a table with θ-values $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2$ in the first quadrant and the corresponding angles in the other quadrants.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Double integrals (Sections 16.1, 16.2)

Name: ______________________ **Section:** ________

Read Sections 16.1 and 16.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Example 1 in Section 16.1.

   (a) What does the function $D = f(x, y)$ represent in this example? (Include units.)

   (b) What do the numbers 0.2, 0.7, 1.2, 1.2, 0.1, ... represent? (Again, include units.)

   (c) What does the factor 750 represent? (Units?)

   (d) What do the products $0.2 \times 750,\ 0.7 \times 750,\ 1.2 \times 750,\ 0.1 \times 750,\ \ldots$ represent? (Units?)

   (e) What does the sum $0.2 \times 750 + 0.7 \times 750 + 1.2 \times 750 + 0.1 \times 750 + \ldots + 1.2 \times 750$ represent? (Units?)

   (f) What are the upper and lower estimates for the fox population? What is the discrepancy between them? How could we improve this?

2. Reread the beginning of Section 16.2, where the fox example is discussed again.

   (a) Write down the expression that gives the exact value of the fox population using nested integrals.

(b) What is the proper name for a nested integral like this?

3. Reread Example 1 in Section 16.2, and try Exercise 5 at the end of the section, page 882.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 16.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercise 1 at the end of the section, page 887.

2. Try Exercise 5 at the end of the section, page 887.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Double integrals in polar coordinates (Section 16.4)

Name: ______________________________ **Section:** __________

Read Section 16.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the subsection entitled "What is dA in Polar Coordinates?" on page 892.

   (a) When using Cartesian (rectangular) coordinates, a grid is made of horizontal and vertical lines. What curves are used to create a grid for polar coordinates?

   (b) When using Cartesian coordinates, the area of a rectangle of width Δx and height Δy is always $\Delta A = \Delta x \cdot \Delta y$, regardless of the position of the rectangle with respect to the origin. Is this true for the areas of the "bent rectangles" in a polar coordinate grid?

   (c) What is the formula for the approximate area of a "bent rectangle" in a polar coordinate grid?

2. Reread Example 3.

   (a) Try Exercises 5 and 6 at the end of the section, page 894.

(b) Try Exercises 7 and 8 at the end of the section, page 894.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Integrals in cylindrical and spherical coordinates (Section 16.5)

Name: ______________________________ **Section:** __________

Read Section 16.5. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the subsections entitled "Cylindrical Coordinates" (pages 896-897) and "Spherical Coordinates" (page 899), and try Exercise 1 at the end of the section, page 901.

2. For Cartesian coordinates the volume element is $dV = dx\,dy\,dz$.

   (a) What is the volume element for cylindrical coordinates?

   $dV \quad =$

   (b) What is the volume element for spherical coordinates?

   $dV \quad =$

3. Look at Exercises 9 and 10 at the end of the section, pages 897 and 898. *Start* these problems by setting up the integrals. You do *not* need to evaluate the integrals.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Change of coordinates in a multiple integral (Section 21.2)

Name: ____________________ **Section:** __________

Read Section 21.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercises 1 and 2 at the end of the section, page 1087.

2. Suppose we have a function $f(x, y)$ which can be expressed in terms of variables s and t using the change of coordinates $x = 2s$ and $y = 3t$. We wish to integrate f over the unit square in the st-plane.

   (a) What is the corresponding region in the xy-plane? What is its area?

   (Hint: The four corners of the unit square in the st-plane are $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$. To find the corresponding points in the xy-plane, use the formulas for the change of coordinates.)

   (b) What is the Jacobian for this change of coordinates?

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 17.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread Examples 4 and 5 in Section 17.1, and try Exercises 7 and 13 at the end of the section, page 923.

2. Reread Examples 2 and 6 in Section 17.1, and try Exercises 21 and 25 at the end of the section, page 923.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 21.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Try Exercises 1-4 at the end of the section, page 1082.

2. Reread Example 2, and try the following exercise: Write a parametrization of the plane through the point $(1, 2, 3)$ and containing the vectors $\vec{v}_1 = \hat{i} + \hat{j} + \hat{k}$ and $\vec{v}_2 = \hat{i} - \hat{j} + 2\hat{k}$.

3. Try Exercises 5 and 6 at the end of the section, page 1082.

4. Reread the subsection "Parameterizations Using Spherical Coordinates," page 1077-1079, and try Exercise 9 at the end of the section, page 1082.

5. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 17.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Give two examples of vector fields arising in physics.

2. Reread Examples 1 and 2, and try Exercise 13 at the end of the section, page 941. Use a table with three x-values and three y-values (as in Example 1) to generate nine vectors to sketch.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Sections 18.1 and 18.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the subsection of 18.1 entitled "What Does the Line Integral Tell Us?" and try Exercises 1-3 at the end of the section, page 964.

2. Name one application of the line integral to physics.

3. Reread Examples 1 and 2 in Section 18.2, and try Exercise 5 at the end of the section, page 972.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Gradient fields and path-independent fields (Section 18.3)

Name: ______________________________ **Section:** ________

Read Section 18.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the Fundamental Theorem of Calculus for Line Integrals (Theorem 18.1) and try Exercise 1 at the end of the section, page 980. (Note that you do not need to calculate a line integral in order to complete the exercise.)

2. (a) Explain, in your own words, what it means for a vector field to be path-independent.

   (b) Explain, in your own words, why we might care about path-independent vector fields.

   (c) Restate, in your own words, what Theorem 18.2 says about path-independent vector fields.

3. Reread Example 3, and try Exercise 13 at the end of the section, page 981.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** __________

Read Section 18.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Explain, in your own words, how to determine whether a vector field is path-independent using circulation.

2. Reread Example 4, and then try Exercise 11 at the end of the section, page 993.

3. **True or false.** Determine whether each statement is true or false, and explain how you know, giving a specific reference (e.g. an example, a theorem, a page number.)

   (a) If $\vec{F}$ is a gradient field with continuous partial derivatives, then its curl is 0.

   (b) If the curl of a vector field $\vec{F}$ is zero, then $\vec{F}$ is path-independent.

(c) If a vector field $\vec{F}$ is undefined at some point, then it cannot be a gradient field.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________ **Section:** ________

Read Section 19.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the beginning of Section 19.1, up to but not including the subsection Example 2.

   (a) Explain, in your own words, what flux means in the context of fluid flow.

   (b) Why is it useful, when computing flux, to represent area by a vector?

   (c) What is the area vector of a disk of radius 2 in the xy-plane oriented upward?

   (d) What is the flux of a constant vector field $\vec{F}(x, y, z) = \vec{v}$ through a flat surface S with area vector $\vec{A}$?

   (e) Try Exercise 16 at the end of Section 19.1, page 1013.

2. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ____ **Section:** ________

Read Section 19.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. In Section 19.2, we calculate flux through three types of surfaces. In each case we need to determine $d\vec{A}$, which can be thought of as the area vector for a small patch of the surface.

   (a) Reread the beginning of Section 19.2, up to but not including Example 1. What is $d\vec{A}$ for a surface that is the graph $z = f(x, y)$, oriented upward?

   (b) Reread the subsection "Flux of a Vector Field Through a Cylindrical Surface" up to but not including Example 2. What is $d\vec{A}$ for a cylinder of radius R centered on the z axis and oriented outward (away from the z-axis)?

   (c) Reread the subsection "Flux of a Vector Field Through a Spherical Surface" up to but not including Example 3. What is $d\vec{A}$ for a sphere of radius R, centered at the origin and oriented outward (away from the origin)?

2. Try Exercise 5 at the end of the section, page 1022.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Flux integrals over parametrized surfaces (Section 21.3)

Name: ______________________________ **Section:** __________

Read Section 21.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the first page of the section.
   (a) Suppose we parameterize a surface by $\vec{r} = \vec{r}(s,t)$, for s,t in some region $\mathcal{R}$ in the st-plane (parameter space). Consider a patch on the surface corresponding to a rectangle in $\mathcal{R}$ with side lengths Δs and Δt.
   (b) Fill in the blanks: "If Δs and Δt are small, the area vector $\Delta\vec{A}$, of the patch is approximately the area vector of the ______________________ defined by the displacement vectors $\vec{v}$ and $\vec{w}$, where $\vec{v}$ = __________ Δs and $\vec{w}$ = __________ Δt. Thus $\Delta\vec{A} \approx$ ______________________."
   (c) What is the formula for $d\vec{A}$ in this case?

2. Try Exercise 1 at the end of the section, page 1091.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, Divergence and the Divergence Theorem (Sections 19.3, 19.4)

Name: ______________________________ **Section:** ________

Read Sections 19.3 and 19.4. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the opening two paragraphs of Section 19.3.

   (a) Explain, in your own words, what the divergence of a vector field is.

   (b) Try Exercise 3 at the end of Section 19.3, page 1030.

2. Practice using the Cartesian coordinate definition of divergence by trying Exercise 7 at the end of Section 19.3, page 1031.

3. Now in Section 19.4, reread the subsection "Calculating the Flux from the Flux Density." Here we have a solid region W in 3-space whose boundary is the closed surface S, and we are interested in finding the total flux of a vector field $\vec{F}$ out of W.

   (a) On one hand, we can calculate the total flux using a flux integral. What is the flux integral representing the total flux out of W?

   (b) On the other hand, we can also use the divergence, which is the flux density. For a small box of volume ΔV, what is the flux out of the box?

(c) Explain, in your own words, why the total flux out of W is the same as the sum of the fluxes of all the small boxes.

(d) Write an integral that represents the total flux out of W using the divergence.

4. Reread Example 1, and try Exercise 7 at the end of the section, page 1037.

5. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, The Curl of a Vector Field (Section 20.1)

Name: ______________________________ **Section:** __________

Read Section 20.1. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Practice using the Cartesian coordinate definition of curl by completing Exercises 3 and 5 at the end of the section, page 1053.

2. Reread Example 2, and try Exercises 11-14 at the end of the section, page 1053.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Name: ______________________________ **Section:** ________

Read Section 20.2. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. Reread the subsection "Calculating the Circulation from the Circulation Density." Here we have an oriented surface S with boundary C, and we are interested in the circulation of a vector field $\vec{F}$ around C.

   (a) On one hand we can calculate the circulation using a line integral. What is the line integral representing the circulation around C?

   (b) On the other hand, we can also use the curl, which is related to the circulation density. For a small piece of the surface with area vector $\Delta\vec{A} = \vec{n}\,\Delta A$, what is the circulation around the boundary of the piece?

   (c) Explain, in your own words, why the circulation around C is the same as the sum of the circulations around each of the small pieces of S.

   (d) Write an integral that represents the circulation of $\vec{F}$ around C using the curl.

2. Reread Example 1, and try Exercise 6 at the end of the section, page 1059.

3. What struck you in reading this section? What is still unclear to you? What questions do you have?

Math 200, The three fundamental theorems (Section 20.3)

Name: ____ **Section:** __________

Read Section 20.3. Take notes in your notebook, making sure to include words and phrases in italics and formulas in blue boxes. Then answer the following questions.

Reading Questions

1. List the three fundamental theorems discussed in this section. In your own words, describe the general pattern into which each of these theorems fits.

2. Practice using the Curl Test by completing Exercises 1 and 2 at the end of the section, page 1065.

3. Practice using the Divergence Test by completing Exercises 7 and 8 at the end of the section, page 1065.

4. What struck you in reading this section? What is still unclear to you? What questions do you have?