1. The motion of a vibrating guitar string is modeled as follows: the vertical displacement, y, of a point on the guitar string at a distance x, measured from one end of the string and time t from the moment the string is plucked, is given by $y = f(x,t) = \cos t \sin(\pi x/10)$, for $0 \leq x \leq 10$.
   (a) Find $f(5,0)$, and explain what it represents in terms of the guitar string.
   (b) Explain what the functions $f(x,0)$ and $f(x,1)$ represent in terms of the guitar string, and sketch graphs of these functions.
   (c) Explain what the functions $f(0,t)$, $f(2,t)$, $f(4,t)$ represent in terms of the guitar string, and sketch graphs of these functions.
   (d) Find $f_t(5,0)$, $f_t(5,\pi/2)$, and $f_t(5,\pi)$, and explain what these numbers represent.
   (e) Find $f_x(0,0)$, $f_x(2,0)$, and $f_x(5,0)$, and explain what these numbers represent.
   (f) Find $f_{xx}(4,0)$, $f_{xt}(4,0)$, $f_{tx}(4,0)$, and $f_{tt}(4,0)$, and explain what these numbers represent.

2. One mole of ammonia gas is contained in a vessel which is capable of changing its volume (a compartment sealed by a poston, for example). The total energy U (in joules), of the ammonia is a function of the volume V (in m^3) of the container, and the temperature T (in K) of the gas.
   (a) Given an initial volume V_0 and temperature T_0, if the volume increases while temperature is held constant, the energy will increase by 840 joules per m^3. Write a mathematical statement in terms of U, V, and/or T that expresses this fact.
   (b) Given an initial volume V_0 and temperature T_0, if the temperature increases while volume is held constant, the energy will increase by 27.32 joules per K. Write a mathematical statement in terms of U, V, and/or T that expresses this fact.
   (c) Combine the two facts in (a) and (b) to write a mathematical statement (involving the differential) about how the energy changes if both volume and temperature are allowed to change.
   (d) Let U be given by a function $f(V,T)$. Find $\text{grad} f(V_0,T_0)$.
   (e) Find a linear approximation for $f(V,T)$ near the point (V_0,T_0).
   (f) Find the directional derivative for f in the direction of $\vec{v} = \hat{i} + 2\hat{j}$.

3. Consider the surface in 3-space that is the solution set to the equation $z + 4 = x^2 + y^2$.
   (a) Sketch the surface, and describe it in words.
   (b) Find a function $f(x,y)$ such that the given surface is the graph of this function. Make a contour diagram for this function, including at least four contours.
   (c) Find a function $F(x,y,z)$ such that the given surface is the level surface of this function corresponding to $F = 2$.
   (d) Three hikers standing on the surface at the point $(1,1,-2)$ notice that their compass points in the direction of steepest ascent. Find a vector (in terms of $\hat{i}$ and $\hat{j}$) that points due north.
   (e) One hiker in (d) chooses to set out in a northerly direction, another chooses to set out due east, and the third sets out in the direction of $\hat{i} + 3\hat{j}$. What are the initial slopes of their chosen paths?
   (f) Find an equation for the plane tangent to the surface at the point $(1,1,-2)$ using the perspective in (b).
   (g) Find an equation for the plane tangent to the surface at the point $(2,0,0)$ using the perspective in (c).

4. Consider $\vec{v} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{w} = 4\hat{i} - \hat{j} + 2\hat{k}$.
   (a) Find two vectors parallel to $\vec{v}$ and two vectors perpendicular to $\vec{v}$.
   (b) Show that $\vec{v}$ and $\vec{w}$ are not parallel and also not perpendicular.
   (c) Resolve $\vec{v}$ into components parallel and perpendicular to $\vec{w}$.
   (d) Find three vectors that are perpendicular to *both* $\vec{v}$ and $\vec{w}$.
   (e) Find the area of the parallelogram whose edges are $\vec{v}$ and $\vec{w}$.