Writing assignment:

Discuss how the dot product (and in some cases the cross product too) are useful for computing the volume of fluid that flows with a constant velocity through a flat surface per unit time.

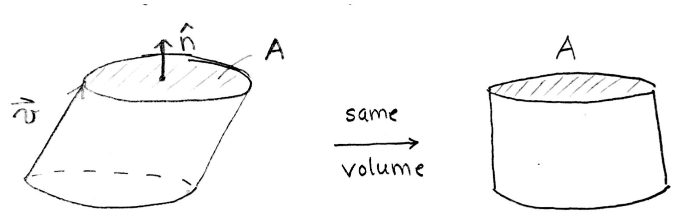
Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Find the magnitude of the component of $\vec{v} = 2\hat{\imath} + \hat{\jmath}$ that is parallel to $3\hat{\imath} + 4\hat{\jmath}$. Illustrate with a picture.

(b) Water is flowing with a velocity of $\vec{v} = 2\hat{\imath} + \hat{\jmath}$ through a flat disc of area $A = 2$ with unit normal vector $\hat{n} = \frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$. What is the volume of water that flows through the surface in one unit of time?

Hint: The volume of water that flows through the surface in one unit of time is the volume of a slanted cylinder as depicted below. This is the same as the volume of the right circular cylinder having the same height as the slanted cylinder. Use (a) to find this height.



(c) The **area vector** $\vec{A} = A\hat{n}$ can be used to describe the area and the orientation of the surface simultaneously. Then we have:

$$A(\vec{v} \cdot \hat{n}) \quad = \quad \vec{v} \cdot (A\hat{n}) \quad = \quad \vec{v} \cdot \vec{A}$$

What is the area vector $\vec{A}$ for the surface in (b)? Check that the formula, $\vec{v} \cdot \vec{A}$, will give you the same value for the volume that you computed above.

(d) Suppose water is flowing with a velocity of $\vec{v} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ through a parallelogram whose edges are given by the vectors $\vec{w}_1 = \hat{\imath} + \hat{\jmath}$ and $\vec{w}_2 = 2\hat{\imath} + \hat{k}$. What is the volume of water that flows through the parallelogram in one unit of time?

Hint: Use the formula: $\vec{v} \cdot \vec{A}$. You will need to find a vector $\vec{A}$ whose magnitude is the area of the parallelogram and whose direction is perpendicular to the plane containing $\vec{w}_1$ and $\vec{w}_2$.