Writing assignment:

Consider the surface S which is the solution set for $xy - z^2 = 0$ in 3-space. Discuss two different ways (i.e. using a graph or a level surface) to find an equation for the plane tangent to the surface at $(1, 4, -2)$. Describe the relationship between the slopes of a plane in the x and y directions and the normal vectors for a plane.

Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Using an appropriate function $F(x, y, z)$, find a vector normal to the surface at $(1, 4, -2)$ and an equation for the plane tangent to the surface at $(1, 4, -2)$.

(b) Using the partial derivatives of an appropriate function $g(x, y)$, find the slopes of the plane in the x and y directions and an equation for the plane tangent to the surface at $(1, 4, -2)$. (Hint: Find a function $g(x, y)$ such that the graph of g in 3-space is the part of the surface S that lies below the xy-plane.)

(c) Supposing a plane has slope m in the x-direction and slope n in the y-direction, find a vector $\vec{n}$ normal to the plane. (Compare your work for (a) and (b), and generalize.)

(d) *Challenge*: Using second partial derivatives for the function $g(x, y)$ from (b), determine whether tangent plane lies above or below the surface (or neither) at $(1, 4, -2)$.