Writing assignment:

Discuss the existence and location of any and all local and global extrema of function $f(x, y) = xy$, first, when the domain is considered to be the whole xy-plane and, second, when the domain is restricted by the constraint $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$.

Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Draw a contour diagram* for $f(x, y)$ with $c = -3, -2, -1, 0, 1, 2, 3$. What shape is the graph of the function in 3-space? Does the function have a global maximum, when its domain is considered to be the whole xy-plane? A global minimum?

(b) Find the critical point(s) of the function $f(x, y) = xy$, and classify it/them using the 2nd derivative test.

(c) Now consider the domain to be the region in the xy-plane given by the constraint $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$. (What does this domain look like? Shade it in on your contour diagram*.) Explain how we know that $f(x, y)$ will attain a global maximum value and a global minimum value on this domain. (You should cite a relevant theorem by name, state what it says, and verify that all the hypotheses of the theorem are satisfied in the present situation.)

(d) Use the method of Lagrange multipliers to find the maximum and minimum values of the function on the domain given by the constraint $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$. (Check that your answers make sense based on your picture from (c).)

*You may use graphing technology to graph the contours for f and the boundary of the restricted domain.