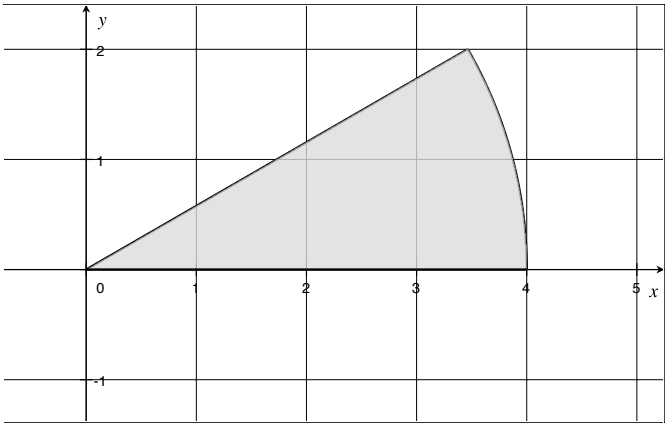
Writing assignment:

Let $\mathcal{R}$ be the planar region shown below, let $f(x, y) = x^2 + y^2$, and consider the double integral $\int_{\mathcal{R}} f\, dA$. Exhibit three different ways to express the integral using iterated integrals, choose one of these iterated integrals to evaluate, and discuss the difficulties that would arise in trying to evaluate the other iterated integrals.



Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Describe the region $\mathcal{R}$ using inequalities in Cartesian coordinates using top and bottom curves to bound y and then left and right endpoints to bound x. Then write $\int_{\mathcal{R}} f\, dA$ as the sum of two iterated integrals in Cartesian coordinates with the inner integrals being with respect to y.

(b) Describe the region $\mathcal{R}$ using inequalities in Cartesian coordinates using left and right curves to bound x and then top and bottom endpoints to bound y. Then write $\int_{\mathcal{R}} f\, dA$ as an iterated integral in Cartesian coordinates with the inner integral being with respect to x. Evaluate the inner integral.

(c) Describe the region $\mathcal{R}$ using inequalities in polar coordinates, write $\int_{\mathcal{R}} f\, dA$ as an iterated integral in polar coordinates, and evaluate the iterated integral.