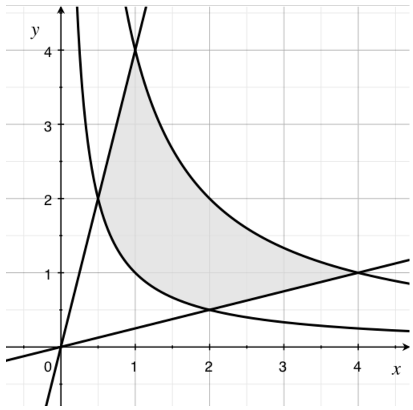
Writing assignment:

Let $\mathcal{R}$ be the planar region in the first quadrant bounded by the curves $y = 4x$, $y = \frac{x}{4}$, $y = \frac{1}{x}$ and $y = \frac{4}{x}$, let $f(x,y) = \frac{x}{y}$, and consider the double integral $\int_{\mathcal{R}} f\,dA$. Explain why this integral would be difficult to evaluate in Cartesian coordinates, choose a change of coordinates that simplifies the integral, and evaluate the integral using your new coordinates.



Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Describe the region $\mathcal{R}$ using inequalities in Cartesian coordinates using top and bottom curves to bound y and then left and right endpoints to bound x. Then write $\int_{\mathcal{R}} f\,dA$ as the sum of three iterated integrals in Cartesian coordinates with the inner integrals being with respect to y.

(b) Describe the region $\mathcal{R}$ using inequalities in Cartesian coordinates using left and right curves to bound x and then top and bottom endpoints to bound y. Then write $\int_{\mathcal{R}} f\,dA$ as the sum of three iterated integrals in Cartesian coordinates with the inner integral being with respect to x.

(c) Show that the boundaries of $\mathcal{R}$ are the lines with equations $xy = 1$, $xy = 4$, $\frac{y}{x} = \frac{1}{4}$ and $\frac{y}{x} = 4$. Use these equations to explain why the change of variables $u = xy$ and $v = \frac{y}{x}$ results in a simpler system of equations for boundary curves in the uv-coordinate plane. Sketch the transformed region $\mathcal{R}'$ in the uv-coordinate plane.

(d) Set up and evaluate a single iterated integral equal to $\int_{\mathcal{R}} f\,dA$ in which you integrate with respect to u and v rather than with respect to x and y. Hint: You may use the fact that

$$\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1$$