Writing assignment:

Consider the two vector fields $\vec{F} = (y-1)\hat{\imath} + x\hat{\jmath}$ and $\vec{G} = x\hat{\imath} + 2x\hat{\jmath}$. Let C_1 be the semicircle given by $x^2 + y^2 = 1$ with $y \geq 0$ and oriented counter-clockwise, and let C be the full unit circle, oriented counter-clockwise. Discuss at least three strategies for evaluating line integrals and how they apply to the following four line integrals:

$$\int_{C_1} \vec{F} \cdot d\vec{r}\,, \qquad \int_{C} \vec{F} \cdot d\vec{r}\,, \qquad \int_{C_1} \vec{G} \cdot d\vec{r}\,, \quad \text{and} \quad \int_{C} \vec{G} \cdot d\vec{r}\,,$$

In particular, discuss the different strategies for conservative vs non-conservative vector fields and over closed paths vs paths with distinct endpoints.

Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Show that $\vec{F} = \vec{\nabla}(xy - x)$, and use the Fundamental Theorem of Line Integrals to evaluate $\int_{C_1} \vec{F} \cdot d\vec{r}$. Can you evaluate $\int_C \vec{F} \cdot d\vec{r}$ without any computation? Explain.

(b) Explain how you know that you cannot use the Fundamental Theorem of Line Integrals to evaluate $\int_{C_1} \vec{G} \cdot d\vec{r}$. Use parametrization to rewrite this line integral as an integral you might find in Calc 2. (You do not need to evaluate this integral.)

(c) Evaluate $\int_C \vec{G} \cdot d\vec{r}$ using Green's Theorem.