Writing assignment:

Consider the vector field $\vec{F} = x^2\hat{i} + z\hat{j} + y\hat{k}$, and let S be the closed cone whose boundary surfaces are $x = \sqrt{y^2 + z^2}$ and the plane $x = 1$. Calculate the flux of $\vec{F}$ out of S directly, i.e. not invoking a theorem. Then describe how the Divergence Theorem can be used to calculate flux in some cases, and apply it as appropriate.

Guide for your work:

A fully complete writing assignment will include the steps outlined below as well as additional discussion of the key ideas and how they are connected.

(a) Use a flux integral to calculate the flux of $\vec{F}$ through the end of the cone in the plane $x = 1$, oriented in the positive x-direction.

(b) Use parameterization and a flux integral to calculate the flux of $\vec{F}$ through the sides of the cone $x = \sqrt{y^2 + z^2}$, oriented outwards from the x-axis, from $x = 0$ to $x = 1$. (Be careful of the orientation!)

(c) What is the total flux of $\vec{F}$ out of the closed cone?

(d) Can you use the Divergence Theorem to calculate the flux in (a), (b), or (c)? State the Divergence Theorem in full, and explain why it applies to one of the fluxes but not the other two.

(e) Use the Divergence Theorem to calculate the flux in (a), (b), or (c), whichever is appropriate, and check that your answer agrees with your previous work.