MATH 221-01, Number Theory & History of Mathematics

MWF 9:00-9:50am, S018

Instructor: Dr. Amy DeCelles

Email: amy.decelles@betheluniversity.edu
Professional Webpage: https://amydecellesmath.org
Office: Science Building 013
Office phone: 7-7095
Office Hours*: M 4-4:50, T 10-10:50, W 12-12:50, R 1-2:50, F 1-1:50

**If you anticipate wanting to come to office hours regularly, and none of these times work for you, please email me about this within the first week of the semester.*

Course Description: Survey of the historical development of mathematics from antiquity through the early twentieth century, with special emphasis on classic mathematical gems from geometry and number theory. Particular focus will be given to the women of mathematics. Mathematical topics will include: prime factorization and the distribution of primes; congruences and residue class arithmetic; quadratic residues and Gauss reciprocity; primality testing and pseudoprimes with applications to cryptography.

Course Prequisite: MATH 132 (Calculus II)

Credits and Workload Expectations: 3 credits, 6-10 hours per week outside the classroom.

Textbook: *Learning Modern Algebra From Early Attempts to Prove Fermat's Last Theorem*, by Al Cuoco and Joseph J. Rotman

Course Objectives:

- A rigorous treatment of the fundamentals of number theory;
- Development and practice of proof techniques common in number theory;
- Increased ability to articulate mathematical arguments; and
- Awareness of the impact of various mathematicians and their contributions.

Program Objectives: This course supports the following math program objectives (as well as similar objectives for Math Education and Engineering programs): M1, M2, M3, M4, M5.

Homework: Homework is assigned in a "rolling trio": reading assignments, discussion problems, and written problems. For example, for Wed Jan 11, you are to write up the solution to a problem from Section 1.1 (which we will have discussed already, in class Mon Jan 9), work on discussion problems for Section 1.2-I (which we will discuss in class on Wed Jan 11), and read and answer questions on Section 1.2-II (which we will discuss in class Fri Jan 13).

Collaboration and Consultation: I encourage you, when working on homework, to collaborate with fellow students, to reread the textbook, and to ask the professor or the Learning Commons tutors for help. You are also free to consult other textbooks or online resources for general

information on the topic. However, *you may not at any point consult any worked solution to an assigned homework problem.* If in doubt about the acceptability of a certain kind of collaboration or consultation, ask the professor. Please see the university policy on academic dishonesty, below.

Attendance: Attendance in class is expected, and a portion of the final grade comes from preparation for and participation in class. Absences for official university functions will be excused, provided that the instructor is notified in advance. Absences due to qualifying family or medical emergencies will also be excused, though the instructor reserves the right to ask for verification. **In the case of an excused absence, it is the student's responsibility to contact the instructor in a timely fashion and make arrangements for an assignment to compensate for the missed discussion participation.**

Late Work: Late work is typically not accepted. The lowest three scores in each assignment category (RQ, D, W) will be dropped at the end of the semester. Extensions on other assignments will be considered if requested before the due date, and work may certainly be submitted before the due date, if arrangements have been made with the professor in advance. If there is a serious, unforeseeable reason for missing more than one week of class, it is the student's responsibility to contact the professor as soon as possible and to make an appointment with the professor upon returning to classes to make a plan, e.g. whether to continue with the course, take an incomplete, or withdraw; and if continuing, how to make up missed work.

Missed Exams: Make-up midterm exams may be given to students with legitimate excuses such as serious illness, university sponsored events, etc., as long as the make-up exam can be taken within a reasonable time frame. If it is not possible to schedule a make-up exam within a reasonable time frame, the grade for the midterm may be prorated from the final exam. Written documentation may be required. Rescheduling the final is not possible except under very extreme circumstances.

Final Course Grade: The overall score for this course will be computed as outlined below. Final letter grades will be assigned based on the overall score, with the two mastery components, written work and exams also being considered separately. In particular, the final letter grade will not be higher than one letter grade above the level of mastery demonstrated on written work or the work on exams. Exceptional performance on the final may also be taken into account.

- Prep and Participation (10%): reading questions (5%), discussion and presentations (5%)
- Written Problems (30%): typically one problem per class, written up nicely
- Quizzes (5%): one or two quizzes per unit
- Midterm Exams (30%): tentatively scheduled for Mon Jan 30, Wed Feb 22, Mon Mar 27
- Final Exam (20%): cumulative; Mon Apr 24, 8-10am
- Best Exam (5%): at the end of the semester, your best exam score will count an extra 5%

Final letter grades will be determined from the overall score as follows:

A	93-100	B+	87-89	B-	80-82	C	73-76	D+	67-69	D-	60-62
A-	90-92	B	83-86	C+	77-79	C-	70-72	D	63-66	F	0-59

Incompletes: Grades of I are normally not given in this course. However, they may be granted due to extenuating circumstances if (i) at least 60% of the course work has been completed at a level of C or better and (ii) the student demonstrates the ability to complete the remaining coursework outside of the classroom. In such cases, a well-documented petition should be submitted to the professor well before grades are due to the Registrar. Please see the university policies on incomplete grades and withdrawing from a class.

Education Majors: Please use the link below to review all appropriate standards.
https://bethelcollege.instructure.com/courses/11416/pages/standards-library

Learning Commons: Located on the lower level of the Miller/Moore Academic Center (AC), the Learning Commons offers in-person and online tutoring services to all Bethel students, including help with any sort of writing projects, from conception to completion. Tutors are trained to give thoughtful feedback and advice on a variety of study skills, understanding concepts pertaining to relevant coursework, and overall writing concerns.

DEI: Bethel University respects the dignity of all God's image-bearers, and stands against racism, prejudice, and discrimination. Because Christ calls us to love our neighbor as ourselves, Christian discipleship includes pursuing the good of those who suffer injustice due to their color, race, or ethnicity. Therefore, we aim to continually transform our classrooms into safe and hospitable spaces where we listen to one another with mercy, learn from and value each other with tenacity, and commit to pursuing justice for the most vulnerable in our community.

Accessibility and Accommodations: Bethel University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, please contact the Center for Academic Success to discuss options. To schedule an appointment, email `rachel.kennedy@betheluniversity.edu` or call 574-807-7460.

Academic Dishonesty: The student handbook (p. 156) states: "Any act of deceit, falsehood or stealing by unethically copying or using someone else's work in an academic situation is strictly prohibited.

1. A student found guilty of plagiarism or cheating will receive an "F" (zero) for that particular paper, assignment or exam. Should this occur, the professor will have an interview with the student and will submit a written report of the incident to the academic dean.

2. If a second offense should occur, the student will be asked to appear before the professor, the academic dean and the vice president for student development.

The student should realize that at this point continuation in a course and even his/her academic career may be in jeopardy. In the event of a recommendation for dismissal, the matter shall be referred to the Student Development Committee."

Cell Phones: Cell phones must be turned off and stowed in book bags during class. Any student using a cell phone for any reason (without permission) will be asked to leave the class and an unexcused absence will be recorded. Students using cell phones during exams or graded activities may be cited for cheating (at professor's discretion). In the case of expected emergencies, students may seek permission from the professor to leave their cell phones on during class, but the phone must remain in the book bag. Professors reserve the right to have operational cell phones in class.

COVID-19: Students are expected to comply with current Bethel University COVID-19 Protocols and Procedures.

Disclaimer: This syllabus is not a legal contract, but serves as a general outline for the semester. The professor reserves the right to announce in advance necessary adjustments to the course as the need arises.

Tentative Schedule: See the next page.

Tentative Semester Schedule

Mon	Wed	Fri
Jan 2, 2023	Jan 4, 2023	Jan 6, 2023
		1.1 Ancient Mathematics
Jan 9, 2023	Jan 11, 2023	Jan 13, 2023
1.1 Ancient Mathematics	1.2 Diophantus-I	1.2 Diophantus-II (Fermat's Last Theorem)
Jan 16, 2023	Jan 18, 2023	Jan 20, 2023
1.2 Diophantus-III (Congruent Numbers)	1.3 Euclid-I (p 20-24) Divisibility, Primes, Division Algorithm	1.3 Euclid-II GCDs, Linear Combinations, Euclid's Lemma, Thm. 1.19
Jan 23, 2023	Jan 25, 2023	Jan 27, 2023
1.3 Euclid-III Euclidian Algorithm I and II	1.4 Nine Fundamental Properties	2.1-I Induction
Jan 30, 2023	Feb 1, 2023	Feb 3, 2023
Exam 1 (Ch 1)	2.1-II Unique Factorization-I	2.1-III Unique Factorization-II
Feb 6, 2023	Feb 8, 2023	Feb 10, 2023
2.2 Binomial Theorem	2.3 Connections (Induction, Fibonacci Numbers)	3.1 Classical Formulas
Feb 13, 2023	Feb 15, 2023	Feb 17, 2023
3.2 Complex Numbers-I	3.2 Complex Numbers-II	3.3 Roots and Powers
Feb 20, 2023	Feb 22, 2023	Feb 24, 2023
3.4 Gaussian and Eisenstein Integers (p 116-122 only)	**Exam 2 (Ch 2-3)**	4.1-I Congruence and Applications (through Example 4.7)
Feb 27, 2023	Mar 1, 2023	Mar 3, 2023
Spring Break	Spring Break	Spring Break
Mar 6, 2023	Mar 8, 2023	Mar 10, 2023
4.1-II Fermat's Little Theorem and b-adic digits	4.1-III Linear Congruences and the CRT (through Example 4.26)	4.1-IV CRT Redux
Mar 13, 2023	Mar 15, 2023	Mar 17, 2023
Crypto	*Crypto*	*Crypto*
Mar 20, 2023	Mar 22, 2023	Mar 24, 2023
Crypto	*Crypto*	*Crypto*
Mar 27, 2023	Mar 29, 2023	Mar 31, 2023
Exam 3 (4.1, Ciphers)	*Crypto*	*Crypto*
Apr 3, 2023	Apr 5, 2023	Apr 7, 2023
Crypto	*Crypto*	Easter Break
Apr 10, 2023	Apr 12, 2023	Apr 14, 2023
Fermat's Last Theorem	*Fermat's Last Theorem*	*Fermat's Last Theorem*
Apr 17, 2023	Apr 19, 2023	Apr 21, 2023
Fermat's Last Theorem	*Fermat's Last Theorem*	*Fermat's Last Theorem*

Unit 1 Plan

Mon	Wed	Fri
Jan 2, 2023	Jan 4, 2023	Jan 6, 2023
		1.1 Ancient Mathematics To do today: • Read syllabus (on Canvas). • Read 1.1. • RQ 1.1; submit on Canvas by 5pm. • Prepare D 1.1 for Monday.
Jan 9, 2023	Jan 11, 2023	Jan 13, 2023
Intro to Course 1.1 Ancient Mathematics Due today: • Read 1.2-I, RQ • D 1.1	1.2 Diophantus-I Due today: • Read 1.2-II, RQ • D 1.2-I • W 1.1	1.2 Diophantus-II Fermat's Last Theorem Due today: • Read 1.2-III, RQ • D 1.2-II • W 1.2-I
Jan 16, 2023	Jan 18, 2023	Jan 20, 2023
1.2 Diophantus-III Connections: Congruent Numbers Due today: • Read 1.3-I, RQ • D 1.2-III • W 1.2-II	1.3 Euclid-I (p 20-24) Divisibility, Primes, Division Algorithm Due today: • Read 1.3-II, RQ • D 1.3-I • W 1.2-III	1.3 Euclid-II GCDs, Linear Combinations, Euclid's Lemma, Thm. 1.19 Due today: • Read 1.3-III, RQ • D 1.3-II • W 1.3-I
Jan 23, 2023	Jan 25, 2023	Jan 27, 2023
1.3 Euclid-III Euclidian Algorithm I and II Due today: • Read 1.4, RQ • D 1.3-III • W 1.3-II	1.4 Nine Fundamental Properties Due today: • Read 2.1-I, RQ • D 1.4 • W 1.3-III	2.1-I Induction Due today: • Read 2.1-II, RQ • D 2.1-I • W 1.4
Jan 30, 2023	Feb 1, 2023	Feb 3, 2023
Exam 1 (Ch 1)	*2.1-II Unique Factorization*	*2.1-III Unique Factzn and p-adic Order*

Assigned Homework Problems

Topic	Discussion	Written
1.1	2, 4(i)(ii), 8, 11, 12, 14	10
1.2-I	21, 22*, 26	27
1.2-II	31*	31*
1.2-III	30, 32, 33*	36
1.3-I	EE1*, 41*, 44	46*
1.3-II	EE2*, 38, 47, 48, Thm 1.19	49*
1.3-III	55(ii), 56, 57, 60, 76(i)	58
1.4	68(i), EE1*, 69*	68(ii)
2.1-I	1, 3, 4*, 5, 8*	7

*An asterisk indicates that a modification or hint has been provided.

EE stands for "Extra Exercise."