Name: ______________________________

Read the introduction to Chapter 4 (the first few paragraphs on page 131) and Section 4.1, through Example 4.7, which ends towards the top of page 136.

Reading Questions

1. The introduction to Chapter 4 provides the context and motivation for what we will study in Chapter 4. Describe this "big picture" in your own words. How does Chapter 4 fit in with what we have studied and what we will study?

2. The first few paragraphs of Section 4.1 provide the motivation for studying congruence. Describe the motivation in your own words.

3. The next several pages give the definition of congruence modulo m and state several results necessary for understanding arithmetic modulo m (Proposition 4.2, Proposition 4.3, Corollary 4.4, and Proposition 4.5.) Write the definition of congruence modulo m in your notebook (word for word), and take notes on the propositions. Which of these are confusing to you?

4. Example 4.1 illustrates the idea of congruence modulo m in daily life. Use this example or an example of your own to understand the statements of the propositions and corollary. Take some notes (in your notebook) on the examples that help you to understand the points that were confusing to you originally. Give an example or two of your own below.

5. Next there are several examples illustrating the usefulness of congruence for solving a variety of problems. Reread Example 4.6 carefully, and try the example proposed by the author at the end of the example.

6. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ____________________________

Read pages 136-140 of Section 4.1.

Reading Questions

1. Use Fermat's Little Theorem (FℓT) to reduce the following numbers modulo 7:
   (a) 777^7
   (b) 70001^{49}
   (c) 700008^6
2. Use Proposition 4.11 to determine whether or not the following numbers are divisible by 3:
   (a) 73741
   (b) 896721
   (c) 895721
3. For any number *not* divisible by 3 in the previous problem, reduce the number modulo 3 (using the note in the How to Think About It box after the proposition).

4. In the explanation of casting out 9s, what does the notation $\Sigma(a)$ mean?

5. Use casting out 9s to reduce 9284654950327 modulo 9.

6. Imitate Example 4.12 to write 1234 in "base 5." (First write in the form $1234 = 5q_1 + r_1$. Then write $q_1 = 5q_2 + r_2$, etc.. Eventually the quotient q_k will be less than 5. I got down to $9 = 5 \times 1 + 4$. At this point start back substituting: substitute $5 \times 1 + 4$ in for the 9 in the previous line ....)

7. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ______________________________

Read pages 141-145 of Section 4.1 (Linear Congruence and the Chinese Remainder Theorem), through the end of Example 4.26.

Reading Questions

1. Under what conditions is the linear congruence $ax \equiv b \bmod m$ solvable?

2. Example 4.19 illustrates the method described in the proof of Theorem 4.17 for solving linear congruences. Reread Example 4.19 carefully to make sure you understand all the steps. Now create your own example of a linear congruence, and solve it. (Make sure that your a, b, and m satisfy the hypotheses in the theorem so that your linear congruence is solvable.)

3. The Chinese Remainder Theorem deals with solutions of systems of linear congruences. Copy that theorem in your notebook here and in your notebook.

4. Create your own example of a system of linear congruences that the Chinese Remainder Theorem would apply to. (Choose m, m', b, b' satisfying the hypotheses of the theorem, and write out the system of linear congruences.) What does the theorem allow you to conclude? (State the conclusion of the theorem in terms of your particular choices of m and m'.) **You do not need to solve the system.**

5. What struck you in this reading? What is still unclear? What remaining questions do you have?

Name: ______________________________

Read pages 145-147 of Section 4.1 (Chinese Remainder Theorem Redux).

Reading Questions

1. In your own words, describe the "localization" technique for solving a system of linear congruences.

2. What is the "important step" in the method for solving a system of linear congruences described at the top of page 147? (Make sure to look back at Exercise 1.56, too.)

3. Explain in your own words what the notation $m_1 m_2 \ldots \widehat{m_i} \ldots m_r$ represents.

4. What role does Exercise 1.58 play in the proof of Theorem 4.27 (CRT Redux)?

5. What struck you in this reading? What is still unclear? What remaining questions do you have?