Math 221, 2.1-I Induction

Name: ______________________________

Reading Questions

1. Give an example of a failure of inductive reasoning.

2. Why is inductive reasoning valuable in mathematics?

3. How is mathematical induction different from inductive reasoning?

4. Give an example of an implication that is true but whose conclusion is false. (See "How To Think About It" box, p 49.)

5. What struck you? What is unclear to you in this reading? What questions do you have?

Math 221, 2.1-II Fundamental Theorem of Arithmetic

Name: ______________________________

Reading Questions

1. State the Fundamental Theorem of Arithmetic and its corollary.

2. Find the prime factorizations of the numbers 480 and 126. Write the prime factorizations in the form given in Theorem 2.10 and in the form given in Corollary 2.11.

3. Find the lcm and gcd of 480 and 126 using prime factorization, as described in Proposition 2.13. How does the form of the prime factorization in this proposition differ from that of Corollary 211?

4. Verify Corollary 2.14 in the case of $a = 480$ and $b = 126$.

5. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ____________________

Reading Questions

1. How does strong induction differ from induction? When is it advantageous?

2. Read the notes on Prime Factorization and p-adic order.

3. For each prime p dividing 126, find the p-adic order of 126, $\mathcal{O}_p(126)$.

4. (a) Calculate $\mathcal{O}_2(25)$, $\mathcal{O}_2(45)$, $\mathcal{O}_2(25 \cdot 45)$, and $\mathcal{O}_2(25 + 45)$.

   (b) Calculate $\mathcal{O}_5(25)$, $\mathcal{O}_5(45)$, $\mathcal{O}_5(25 \cdot 45)$, and $\mathcal{O}_5(25 + 45)$.

5. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ______________________________

Reading Questions

1. Verify that the sixth row of Pascal's triangle can be obtained from the fifth row, as described on the top of p 64.

2. State Pascal's Theorem (Proposition 2.24).

3. Verify that the numbers in the sixth row of Pascal's triangle satisfy the formula given in Pascal's Theorem.

4. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ______________________________

Reading Questions

1. Explain in your own words what an inductive (or recursive) definition is.

2. What is the golden ratio?

3. How is the golden ratio connected to the Fibonacci sequence?

4. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ______________________________

Reading Questions

1. Verify that the three cube roots of unity (1, ω, and $\bar{\omega}$) are solutions of $x^3 - 1 = 0$.

2. What are the three cube roots of 8?

3. When we are interested in solutions to the general cubic equation, $aX^3 + bX^2 + cX + d = 0$, it suffices to consider the case $a = 1$, $b = 0$. Why is this sufficient?

4. In what sense is Example 3.4 "good" and Example 3.5 "bad"?

5. True or false: the procedures for finding roots of the general cubic and the general quartic can be extended to find roots of the general quintic.

6. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ____________________

Reading Questions

1. Sum up Proposition 3.8 in one sentence. (See the text immediately preceding the proposition.)

2. Find the reciprocal of the complex number $3 + 4i$ using the formula on the top of page 96, and check your answer by multiplying by $3 + 4i$ to get 1.

3. Find the reciprocal of $3 + 4i$ using the complex conjugate, as in Proposition 3.11.

4. Cite the relevant parts of propositions in this section that correspond to the nine fundamental properties of the real numbers described in Section 1.4. (Include the names of the properties.)

5. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ____

Reading Questions

1. A vector has *magnitude* and *direction*. What are the corresponding geometric properties of a complex number? (List multiple names for the same property, if multiple names are given.)

   Given a complex number z, how does one find these two properties?

2. Describe the geometric interpretation of multiplication of complex numbers in one sentence.

3. Let $z = \sqrt{3} + i$ and $w = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$. Find zw in two ways: (a) using the definition of multiplication on page 94 and (b) converting z and w to polar form and using Theorem 3.18.

Recall: $z = \sqrt{3} + i$ and $w = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$. Above you found zw in two ways. Verify that the two are the same by converting your result from (a) to polar form. Illustrate with a picture of z, w and zw in the complex plane.

Consider an arbitrary nonzero complex number w', and let z be as above. Where is zw' located in the complex plane, relative to w'? Illustrate with a picture.

4. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ______________________________

Reading Questions

1. State Euler's theorem (Theorem 3.25).

2. State De Moivre's Theorem, in both forms (Theorem 3.20 and Corollary 3.27).

3. Write the complex roots of $x^{12} - 1$ in exponential polar form, and draw them in the complex plane. Which of these are primitive?

   What is $\phi(12)$?

   If ζ denotes one of the complex roots of $x^{12} - 1$, what is ζ^{49}?

4. What struck you? What is unclear to you in this reading? What questions do you have?

Name: ______________________________

Note. *Read the following three parts of Section 3.4: Norms (p 116-117), Gaussian Integers: Pythagorean Triples Revisited (p 119-120), and Eisenstein Integers (p 120-121 up to but not including the definition of an Eisenstein triple).*

Reading Questions

1. State the definition of the norm of a complex number.

2. State the definition of the Gaussian integers.

3. State the definition of the Eisenstein integers.

4. What struck you? What is unclear to you in this reading? What questions do you have?