Name: _____

Read Section 10.1, on normal subgroups and factor groups.

Reading Questions

1. Make sure you know the definition of normal subgroup, the equivalent conditions given in Theorem 10.3, and the definition of a factor/quotient group.

2. Let $G = D_4$, the group of symmetries of the square, and let $H = \langle r \rangle$, the rotation subgroup.

   (a) Verify that H is normal in G by writing out the left and right cosets of H in G .

   (b) Write out a Cayley table for the factor group G/H.

3. Explain why $4\mathbb{Z}$ is a normal subgroup in $\mathbb{Z}$, and write out a Cayley table for the factor group $\mathbb{Z}/4\mathbb{Z}$.

4. True or false, with explanations.

   (a) Every subgroup of an abelian group is normal.

(b) A subgroup H of a group G is normal if the number of left cosets of H in G equals the number of right cosets of H in G.

(c) A subgroup H of a group G is normal if there is an element $g \in G$ such that $gH = Hg$.

(d) If H is a normal subgroup of a group G, then $|G/H| = [G : H]$.

5. What struck you in this reading? What is still unclear? What remaining questions do you have?