Name: _____

Read Section 15.2, up to and including Example 15.17.

Note 1. A *simple group* is one with no proper nontrivial normal subgroups.

Note 2. The following is useful for understanding Example 15.12.

> **Lemma.** Let G be a group and H, K be normal subgroups in G with $H \cap K = \{e\}$. Then $hk = kh$ for all $h \in H, k \in K$. Thus, $HK \cong H \times K$.

Note 3. Theorem 15.13 can be strengthened by adding the following, "Further, if H is a normal subgroup of G and G/H is abelian, then G' is a subgroup of H."

Reading Questions

1. Use Theorem 15.10 to find three composite numbers n for which the following is true: If G is a group of order n, then G is cyclic.

2. Reread Example 15.14 carefully.

   (a) Explain how the Third Sylow Theorem implies that G has only one subgroup of order 47.

   (b) Explain why the factor group G/H_1 must be abelian.

(c) Explain why the order of the commutator subgroup G' of G must be 1 or 47.

(d) Explain why we are done if $|G'| = 1$.

(e) Explain how we know that $|G'| \neq 47$.

3. What struck you in this reading? What is still unclear? What remaining questions do you have?