

Name: _____

Reading Questions

1. Make sure you know the definitions of the following terms: ring homomorphism, ring isomorphism, kernel of ring homomorphism, ideal, trivial ideal, principal ideal, and quotient/factor ring.
2. Make sure you know the properties of ring homomorphisms (Proposition 16.22, 16.27) and the statements of the ring isomorphism theorems.
3. *Reduction modulo n .* Consider the ring homomorphism $\phi : \mathbb{Z} \rightarrow \mathbb{Z}_n$ by $a \mapsto a \bmod n$.
 - (a) What is the kernel of ϕ ?
 - (b) What is the image of ϕ ?
 - (c) What does the First Isomorphism Theorem allow us to conclude in this case?
4. *Evaluation at α .* Consider the ring homomorphism $\phi : C[a, b] \rightarrow \mathbb{R}$ by $f \mapsto f(\alpha)$, where $\alpha \in [a, b]$.
 - (a) What is the kernel of ϕ ?
 - (b) What is the image of ϕ ?
 - (c) What does the First Isomorphism Theorem allow us to conclude in this case?

5. True or false, with reasons.

(a) If $\phi : R \rightarrow S$ is a ring homomorphism, then the image of ϕ is an ideal in S .

(b) If $\phi : R \rightarrow S$ is a ring homomorphism, then the kernel of ϕ is an ideal in S .

(c) If $\phi : R \rightarrow S$ is a surjective ring homomorphism, then $R/\ker\phi \cong S$.

(d) If S is a subring of R , then the cosets of S in R form a subring of R called the factor ring R/S .

6. What struck you in this reading? What is still unclear? What remaining questions do you have?