Name: ____

Reading Questions

1. Make sure you know the definitions of maximal ideal and prime ideal as well as the equivalent characterizations of them in a commutative ring with identity (Theorem 16.35 and Proposition 16.38).

2. True or false, with reasons.

   Let R be a commutative ring with identity.

   (a) An ideal M is maximal in R if and only if the only ideal containing M is R.

   (b) An ideal P is prime in R if and only if the following implication is true for any a, b in R: if $ab \in P$, then $a \in P$ or $b \in P$.

   (c) An ideal M in R is maximal if and only if R/M is a field.

   (d) An ideal P in R is prime if and only if R/P is an integral domain.

   (e) If P is a prime ideal in R, then P is a maximal ideal in R.

3. Reread the proof of Theorem 16.35.

   (a) In the first part of the proof (before the "Conversely ..." part), we construct a certain ideal I and prove $I = R$. How do we know that I is *properly* contains M, i.e. how can we be sure $I \neq M$?

   (b) In the second part of the proof (beginning with the "Conversely ..."), how do we know that R/M has at least two elements?

   (c) How do we know that if $1 \in I$, $I = R$?

4. What struck you in this reading? What is still unclear? What remaining questions do you have?