

Name: \_\_\_\_\_

### Reading Questions

1. Make sure you know the definitions of the following terms: polynomial over a ring  $R$  with indeterminate  $x$ , coefficients, leading coefficients, monic polynomial, degree of a polynomial, addition and multiplication of polynomials, zero/root of a polynomial, gcd of two polynomials, relatively prime polynomials, evaluation homomorphism.
2. Make sure you know the statements of the main results: Theorem 17.3 and Proposition 17.4 on the structure of polynomial rings, Theorem 17.6 and Proposition 17.10 on the “arithmetic” of a polynomial ring over a *field*, and Corollary 17.8 and Corollary 17.9 about roots/zeros of polynomials.
3. True or false, with reasons.
  - (a) The degree of the zero polynomial is zero.
  - (b) Let  $R$  be a commutative ring with identity and  $p, q \in R[x]$ . Then  $\deg(pq) = \deg(p) + \deg(q)$ .
  - (c) Let  $R$  be a commutative ring with identity,  $\alpha \in R$ , and  $\phi_\alpha$  the evaluation homomorphism  $\phi_\alpha : R[x] \rightarrow R$ . The kernel of  $\phi_\alpha$  consists precisely of polynomials in  $R[x]$  having  $\alpha$  as a root.
  - (d) Let  $F$  be a field,  $\alpha \in F$ , and  $\phi_\alpha$  the evaluation homomorphism  $\phi_\alpha : F[x] \rightarrow F$ . The kernel of  $\phi_\alpha$  consists precisely of polynomials in  $F[x]$  having  $x - \alpha$  as a factor in  $F[x]$ .
  - (e)  $\mathbb{Q}[x]$  is an integral domain but not a field.

4. What struck you in this reading? What is still unclear? What remaining questions do you have?