Name: ____

Reading Questions

1. Make sure you know the definition of an irreducible polynomial, the results about factoring/irreducibility of polynomials over $\mathbb{Q}$ (Gauss' Lemma: Theorem 17.14, Corollary 17.15, and Eisenstein's Criterion: Theorem 17.17), and study the proofs of Theorems 17.20 and 17.22.

2. **Note.** There is a typo in the proof of Theorem 17.22. The second-to-last sentence should read, "If $g(x)$ is constant, then $f(x)$ is a constant multiple of $p(x)$ and $I = \langle p(x) \rangle$."

3. State the contrapositive of Gauss' Lemma. (This is often more useful than Gauss' Lemma itself.)

4. Use Corollary 17.15 to show that $1 + x + x^3$ is irreducible over $\mathbb{Q}$.

5. Use Eisenstein's Criterion to construct a quintic polynomial irreducible over $\mathbb{Q}$.

6. Reread the proof of Theorem 17.22.

   (a) The third sentence says, "Since a maximal ideal must be properly contained inside of $F[x]$, $p(x)$ cannot be a constant polynomial." Explain why this is so.

(b) A little later, it is asserted that if $f(x) \in \langle p(x) \rangle$, then $f(x)$ is a multiple of $p(x)$. Why is this?

(c) In the second part of the proof ("Conversely ..."), it is shown that if $p(x) \in I = \langle f(x) \rangle$, then $p(x) = f(x)g(x)$, where $f, g \in F[x]$. How do we know that one of $f(x), g(x)$ must be constant? If $f(x)$ is a constant, what is I? If $g(x)$ is constant, what is I?

7. What struck you in this reading? What is still unclear? What remaining questions do you have?