Name: _____

Reading Questions

1. Make sure you know the definitions of the following terms: extension field, base field, algebraic element, algebraic extension, transcendental element, transcendental extension, minimal polynomial, the degree of an algebraic element, finite extension, and the degree of a finite extension.

2. Make sure you are familiar with the statements of the theorems, especially Theorems 21.5, 21.10, 21.13, and 21.17.

3. Give an example of an algebraic element over $\mathbb{Q}$ and an example of a transcendental element over $\mathbb{Q}$.

4. Consider $\mathbb{R}/\mathbb{Q}$, and let $\alpha = \sqrt[3]{5} \in \mathbb{R}$.

   (a) What is the minimal polynomial of α over $\mathbb{Q}$? What is the degree of α over $\mathbb{Q}$?

   (b) Give an example of another polynomial in $\mathbb{Q}[x]$ that has α as a root.

   (c) According to Theorem 21.13, every element of $\mathbb{Q}(\sqrt[3]{5})$ can be written in what form?

   (d) What is the degree of $\mathbb{Q}(\sqrt[3]{5})$ over $\mathbb{Q}$?

5. What struck you in this reading? What is still unclear? What remaining questions do you have?