Name: ______________________________

Reading Questions

1. Make sure you know the definitions of a splitting field for a polynomial and a polynomial that splits over a field and that you understand the statements of Theorem 21.31 and Corollary 21.24 (existence and uniqueness–up to isomorphism–of splitting fields).

2. Reread Example 21.29. Explain why $\mathbb{Q}(\sqrt{2}, i)$ is a splitting field for $p(x)$.

3. Reread Example 21.30. Find a splitting field for $p(x)$, and explain why $\mathbb{Q}(\sqrt[3]{3})$ is *not* one.

   Hint: Recall that the complex cube roots of 1 are 1, ω, and ω^2, where $\omega = \frac{-1+\sqrt{3}i}{2}$ and $\omega^2 = \frac{-1-\sqrt{3}i}{2}$.

4. True or false, with reasons.
   Let E be an extension of a field F and $p(x) \in F[x]$ of degree n.
   (a) The extension field E is a splitting field $p(x)$ if $p(x)$ factors into linear factors in $E[x]$.

   (b) If $p(x)$ has n distinct roots in E, then it splits over E.

   (c) If E and K are splitting fields for $p(x)$ over F, then there is an isomorphism $\phi : E \to K$ such that $\phi(F) = F$.

5. What struck you in this reading? What is still unclear? What remaining questions do you have?