Name: ____

Reading Questions

1. Make sure you understand what it means for a field extension to be an *extension by radicals* and for a polynomial to be *solvable by radicals*. Also make sure you know the main theorem (Theorem 23.29), how field extensions involving roots of unity are used in the proof of the main theorem, and how it the main theorem is applied to prove the insolvability of the quintic (Example 23.32.)

2. Reread the proof of Lemma 23.27. Let ω be a complex n^{th} root of unity. (This notation is different from the notation we used in Abstract Algebra I, but it is the notation used in the proof.)

   True or false:

   (a) The Galois group of $\mathbb{Q}(\omega)/\mathbb{Q}$ is abelian.

   (b) The Galois group of $x^n - a$ is abelian for any $a \in \mathbb{Q}$.

3. Reread Example 23.32.

   (a) How do we know $f(x) = x^5 - 6x^3 - 27x - 3$ is irreducible over $\mathbb{Q}$?

   (b) How do we know that $f(x)$ has exactly 3 distinct real roots?

(c) Let K be the splitting field of f over $\mathbb{Q}$. How do we know $G = \mathrm{Gal}(K/\mathbb{Q})$ is a subgroup of S_5?

(d) Let α be a root of f in E. What is the degree of $\mathbb{Q}(\alpha)/\mathbb{Q}$? What does this imply about the order of G?

(e) How do we know that G contains a 5-cycle? (Hint: Use (d) and Cauchy's theorem.)

(f) How do we know that G contains a transposition?

(g) How do we know $f(x)$ is not solvable by radicals?

4. What struck you in this reading? What is still unclear? What remaining questions do you have?